Lecture 23

Plotting Functions of Two Variables

Functions on Rectangular Grids

Suppose you wish to plot a function $f(x, y)$ on the rectangle $a \leq x \leq b$ and $c \leq y \leq d$. The graph of a function of two variables is of course a three dimensional object. Visualizing the graph is often very useful.

For example, suppose you have a formula

$$f(x, y) = x \sin(xy)$$

and you are interested in the function on the region $0 \leq x \leq 5$, $\pi \leq y \leq 2\pi$. A way to plot this function in MATLAB would be the following sequence of commands:

```
≫ f = @(x,y) x.*sin(x.*y)
≫ [X,Y] = meshgrid(0:.1:5,pi:.01*pi:2*pi);
≫ Z = f(X,Y)
≫ mesh(X,Y,Z)
```

This will produce a 3-D plot that you can rotate by clicking on the rotate icon and then dragging with the mouse. Instead of the command `mesh`, you could use the command

```
≫ surf(X,Y,Z)
```

The key command in this sequence is `[X Y] = meshgrid(a:h:b,c:k:d)`, which produces *matrices of x and y values* in `X` and `Y`. Enter:

```
≫ size(X)
≫ size(Y)
≫ size(Z)
```

to see that each of these variables is a 101×51 matrix. To see the first few entries of `X` enter

```
≫ X(1:6,1:6)
```

and to see the first few values of `Y` type

```
≫ Y(1:6,1:6)
```

You should observe that the x values in `X` begin at 0 on the left column and increase from left to right. The y values on the other have start at π at the top and increase from top to bottom. Note that this arrangement is flipped from the usual arrangment in the x-y plane.

In the command `[X Y] = meshgrid(a:h:b,c:k:d)`, h is the increment in the x direction and k is the increment in the y direction. Often we will calculate

$$h = \frac{b-a}{m} \quad \text{and} \quad k = \frac{d-c}{n},$$

where m is the number of *intervals* in the x direction and n is the number of intervals in the y direction. To obtain a good plot it is best if m and n can be set between 10 and 100.

A common way of visualizing a function of two variables is by a *Contour Plot*. In a contour plot we draw several *level curves* of the function, which are the curves at which the function is equal to a few values. A topographical map is an example of a contour plot. To produce a contour plot for the function $f(x, y)$ as above, since we have input the function itself and created a meshgrid on which we want to plot it, we simply input:

```
≫ contour(X,Y,Z,10)
```

The optional number "10" specify how many contour curves to display.

For another example of `meshgrid`, `contour` and `mesh`, try the following and look at `X` and `Y`.

```
≫ [X,Y] = meshgrid(0:.05:4,1:.02:2);
≫ Z = (X+Y)./(1+X.^2+Y.^2);
≫ contour(X,Y,Z,11)
≫ mesh(X,Y,Z)
```

Scattered Data and Triangulation

Often we are interested in objects whose bases are not rectangular. For instance, data does not usually come arranged in a nice rectangular grid; rather, measurements are taken where convenient.

In MATLAB we can produce triangles for a region by recording the coordinates of the vertices and recording which vertices belong to each triangle. The following script program produces such a set of triangles:

```
% mytriangles
% Program to produce a triangulation.
% V contains vertices, which are (x,y) pairs
V = [ 1/2 1/2  ;  1   1 ; 3/2 1/2 ;  .5  1.5 ;  0    0
       1    0  ;  2   0 ; 2    1  ; 1.5 1.5  ;  1    2
       0    2  ;  0    1]
% x, y are row vectors containing coordinates of vertices
x = V(:,1)';
y = V(:,2)';
%  Assign the triangles using Delaunay's algorithm
T = delaunay(x,y)
```

You can plot the triangles using

```
≫ trimesh(T,x,y)
```

You can also prescribe values (heights) at each vertex directly (say from a survey):

```
≫ z1 = [ 2 3 2.5 2 1 1 .5 1.5 1.6 1.7 .9 .5 ];
```

or using a function:

```
≫ f = @(x,y) abs(sin(x.*y)).^(3/2);
≫ z2 = f(x,y);
```

The resulting profiles can be plotted:

```
≫ trimesh(T,x,y,z1)
≫ trisurf(T,x,y,z2)
```

Each row of the matrix `T` corresponds to a triangle, so `T(i,:)` gives triangle number `i`. The three corners of triangle number `i` are at indices `T(i,1)`, `T(i,2)`, and `T(i,3)`. So for example to get the y-coordinate of the second point of triangle number 5, enter

```
≫ y(T(5,2))
```

To see other examples of regions defined by triangles, download `mywedge.m` and `mywasher.m` and run them. Each of these programs defines vectors `x` and `y` of x and y values of vertices and a matrix `T`. As before `T` is a list of sets of three integers. Each triple of integers indicates which vertices to connect in a triangle.

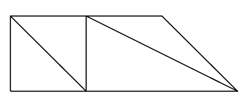
To plot a function, say $f(x,y) = x^2 - y^2$ on the washer figure try

```
≫ mywasher
≫ z = x.^2 - y.^2
≫ trisurf(T,x,y,z)
```

Note again that this plot can be rotated using the icon and mouse.

Exercises

23.1 Plot the function $f(x,y) = \sin(x)\, e^{-x^2-y^2}$ on the rectangle $-3 \le x \le 3$, $-2 \le y \le 2$ using `meshgrid` and `mesh`. Make an appropriate choice of m and n and if necessary a rotation to produce a good plot. Calculate the h and k corresponding to your m and n. Turn in your plot and the calculation of h and k.

23.2 Modeling after `mywasher.m`, produce  using integer coordinates for the vertices. Use the `axis` command to zoom out so the outside edges are clearly visible. Compute $z = 3x + y^2$ and plot the graph. Turn in your program and the plots.