Graphical Representation of a Geminal Configuration

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Introduction

- For a given geminal configuation computing the expanded determinant is a tedious task
- Hence in order to compute the determinants in an easier way, we represent the geminal configuration graphically

Some examples: Geminals represented graphically

1.  corresponds to

$$\int \sum_{v=1}^{N} \sum_{n} \sum_{a=1}^{N} \sum_{b=1}^{N} \sum_{m=1}^{N} \int \left(\int w_1(|\gamma - \gamma'|) \phi_a(\gamma) \theta_v(\gamma) d\gamma \right) \phi_b(\left(\int w_2(|\gamma'' - \gamma'''|) \phi_m(\gamma'') \theta_b(\gamma'') d\gamma'' \right) \sum_u \int \left(\int w_3(|\tilde{\gamma} - \tilde{\tilde{\gamma}}|) \phi_u(\tilde{\gamma}) \theta_n(\tilde{\gamma}) d\tilde{\gamma} \right) \phi_v(\tilde{\tilde{\gamma}}) \theta_a(\tilde{\tilde{\gamma}}) d\tilde{\tilde{\gamma}} \theta_u(\gamma''') \phi_n(\gamma''') d\gamma'''$$

1.  corresponds to

$$\sum_{a=1}^{N} \sum_{b=1}^{N} \sum_{m=1}^{N} \sum_{n=1}^{N} \sum_{u=1}^{N} \sum_{v=1}^{N} \int \int \int \int \int \int \omega_1(|\gamma - \gamma'|) \omega_2(|\gamma'' - \gamma'''|) \omega_3(|\tilde{\gamma} - \tilde{\tilde{\gamma}}|) \phi_a(\gamma) \phi_b(\gamma') \phi_m(\gamma'') \phi_n(\gamma''') \phi_u(\tilde{\gamma}) \phi_v(\tilde{\tilde{\gamma}}) \theta_a(\tilde{\gamma}) \theta_v(\gamma) \theta_b(\gamma'') \theta_m(\gamma') \theta_n(\tilde{\tilde{\gamma}}) \theta_u(\gamma''') d\gamma \, d\gamma' \, d\gamma'' \, d\gamma''' \, d\tilde{\gamma} \, d\tilde{\tilde{\gamma}}$$

Graphical Represention:Components of a Geminal Configuration

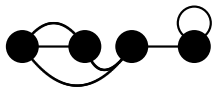
In a given geminal configuration

- Each **Variable** is represented by a node,
- **Geminals** are represented as a line connecting to the two nodes, 
- **Summation Indexes** are represented as curves, 

Graphical Representation cont'd

- Hence, for a given geminal configuration such as  the exapansion of it's determinant will lead all graphs
- Each graph is determined on the basis that each variable ● has exactly two curved edges coming out from it
  - *Loop* on a node or,
  - Edges of two *Curves* on a node
- We disregard some graphs from all the graphs considered by all combinations of loops and curves possible due to the symmetry of graphs (Which we consider them as duplicates)

Symmetry of Graphs

- Consider the following two graphs
  - Figure 1: 
  - Figure 2: 
    If the first and second nodes are switched in "Figure 1" we will obtain "Figure 2" . Due to the symmetry of the graphs we consider only one of the figures
- Also if,
  - Figure 3: 

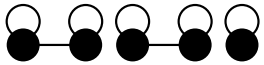
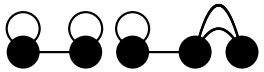
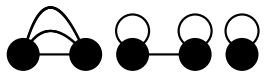
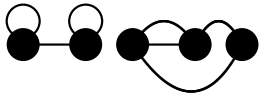
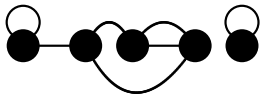
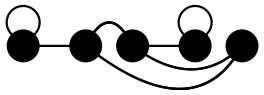
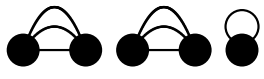
  is compared with "Figure 2", both figures give the same graph due to Symmetry. i.e. Figure 3 can be obtained by switching the node pair(1,2) with the node pair (3,4) in Figure 2
- Hence we consider only the distinct graphs (considering symmetry) in determining all possible graphs

Objective

- Determine all the distinct graphs(considering symmetry) that can be represented that corresponds to the expansion of the determinant
- To implement all the graphs that were determined for each geminal configuration in LaTeX

Example

For the geminal configuration of  there are 21 distinct graphs in total

- 
- 
- 
- 
- 
- 
- 
- 
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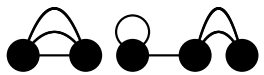
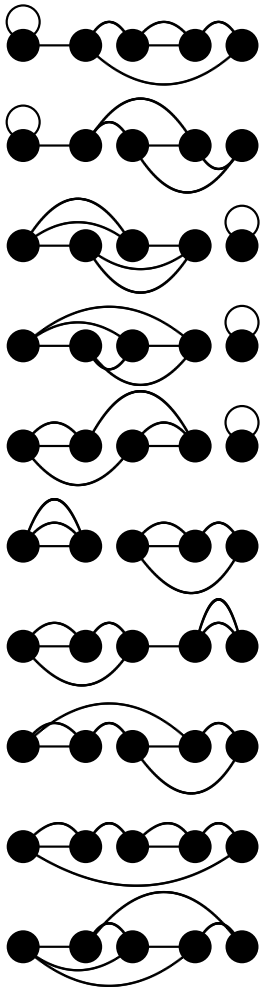
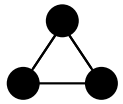
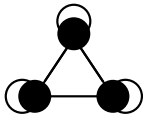
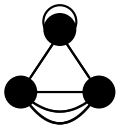
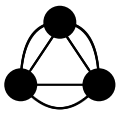
- 

Example continued

- 
- Find the missing graph and you will recieve a gift!

Another Example

- Distinct graphs for the geminal configuration of 

1. 
2. 
3. 

Implementing Graphs in LaTeX

- Examples of some codes to implement graphs in LaTeX
  - **A node**

    ```
    \put(0,0){\circle*{2}}
    ```

    will give a filled circle at the coordinate (0,0) with a diameter of 2
  - **A loop on a node**

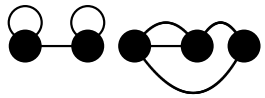
    ```
    \put(0,1.5){\circle{2}}
    ```

    will give a circle at the coordinate (0,1.5) with a diameter of 2
  - **A line between two nodes**

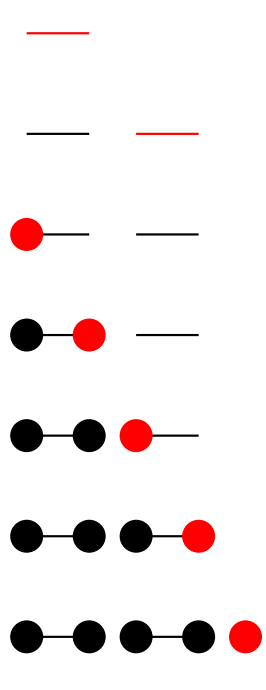
    ```
    \put(0,0){\line(1,0){4}}
    ```

    will give a line of 4 units long starting at the coordinate (0,0) with run of 1 and rise of 0

Example: Code used in LaTeX to generate graphs

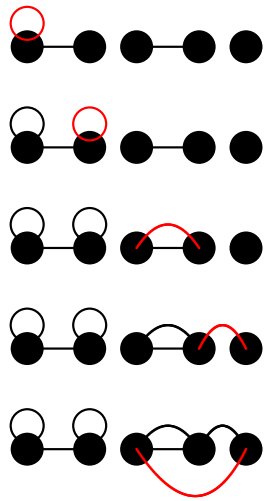
Code for graphing  is described below

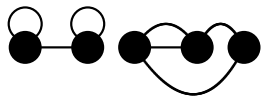
```
\newcommand{\pictggDisjointVaabbmnnuum}{
\setlength{\unitlength}{1mm}
\begin{picture}(19,5)(0,-2)
\put(0,0){\line(1,0){4}}
\put(7,0){\line(1,0){4}}
\put(0,0){\circle*{2}}
\put(4,0){\circle*{2}}
\put(7,0){\circle*{2}}
\put(11,0){\circle*{2}}
\put(14,0){\circle*{2}}
```



Code of the graph-cont'd

```
\put(0,1.5){\circle{2}}
\put(4,1.5){\circle{2}}
\qbezier(7,0)(9,3)(11,0)
\qbezier(11,0)(12.5,3)(14,0)
\qbezier(7,0)(11,-6)(14,0)
\end{picture}
}
```



Hence we finally obtain 

Conclusion

- We think we have found all graphs for different geminal configurations with 3 nodes, 4 nodes and 5 nodes
- In each case we have eliminated most of the graphs due to symmetry giving duplicate graphs and found all possible distinct graphs
- We also learnt to implement graphs in LaTeX which was new to us having worked only with documents involving math equations

Thank you!