Journal of Rob Vanyo for Spring 2008

Rob Vanyo

Spring Quarter 2008

Week of April 9, 2008

For this quarter, I will try to prove that the tails of the sequences approaching the target points in my plots from last quarter are lines in the 2-dimensional plane depicted in the pictures. Thus, I must pick a generalized rank-3 target point, apply the central-difference formula of the identity I've been working with, create a sequence of approximations of the target, and then apply the methods used to transform these rank-2 points into xy-coordinates by hand, and show that a change in h results in a linear change in the xy-plot. I have not made much progress with this as of right now, but I intend to puzzle out some tensor properties and nuances of the identity formula, as well as the reasons behind why the plotting method works, in the coming week. I think my weakness here is just with my minimal knowledge and understanding of tensors, tensor products, and the various properties of tensor products.

I will also be presenting a poster at the research and creative activity expo this quarter, it will detail my work of winter quarter as well as any and all accomplishments this quarter. The proof that the tails in the images are lines will be a part of that poster.

Week of April 16, 2008

This past week I've been working by hand to try and prove that the points plotted as the rank-2 sequence whose limit is a rank-3 point in the plane are on a line. Recall that this sequence of rank-2 points is given by applying the identity I have been working with to the rank-3 target point, and converting it into a rank-2 point dependent on a parameter h. The sequence of rank-2 points approaching the target is then achieved by substituting various values of h that approach zero into the rank-2 point.

The work by hand has been ugly, but I started with a generalized rank-3 target point that is on the plane of the three specific basepoints I have been working with. This was achieved by making the target a weighted sum of the basepoints. I then applied the identity to the target and found a rank-2 point in terms of h that could be used to define the sequence of points approaching the target. I then began to convert the rank-2 point into an xy-point based on the methods used by the Sets Group last fall. Converting a rank-r point into an xy-point consists partly of finding the three coefficients that can be multiplied onto the basepoints of the plane to create a projection of the rank-r point onto the plane. With a rank-3 point that is already known to be on the plane, this would be trivial, however, with a rank-2 point, it is not. However, I did find the scaled coefficients required, and this is where I stopped. With these coefficients, and the knowledge that some other values required to create x and y are not dependent on h, I have roughly found x and y in terms of h. However, the functions $x(h)$ and $y(h)$ both involved h^2, and don't immediately seem like we'll have a linear result. However, in this case I found that it is no problem to find the limit of these coordinates as h approaches zero. I could simply substitute zero for h and see clearly that the xy-coordinates plotted are exactly the coordinates that would be found had I converted the rank-3 target point into an xy-point, instead of the rank-2 sequence point.

I then did try to see if there was a linear relationship between x and y by solving for h in terms of y and then substituting $h(y)$ into $x(h)$ to see if that gives the equation of a line. It certainly seems to, although it is a mess. In the next week, I plan to clean it up, and also explore the relationship between h and the angle between the rank-2 point and the plane (what is used to determine the color of the plotted points).

Week of April 23, 2008

Last week I was able to show that the xy-points of the tail of a sequence that approaches a rank-3 target do lie on a line. This past week, I have been trying to show that the lines these tails lie on all intersect at the same point. To this end, I have found y-values for two general rank-3 targets, in terms of x, constants generated from the basepoints, and two pairs of weights on the basepoints (since putting weights on the basepoints is how rank-3 targets are chosen, and the third weight is always dependent on the other two, so only two are needed). I then set them equal, and solved for x in terms of the two pairs of weights, and the various constants calculated from the basepoints. If all the lines will intersect at the same point, then the x-coordinate should not depend on the weights, since the intersect point should be independent of which two lines I choose, or equivalently, which two target points are approached. Unfortunately, what I have now is a factoring nightmare. Basically, I am stuck trying to factor a very ugly fraction into something that can divide out any terms or factors that involve any of the four weights, thus making x depend solely on the basepoints. My plan for now is to keep on trying to factor this hideous thing:

$$x = \frac{KCa_1' - 3KCa_1'a_2 + KCa_2' - 3Ka_2'a_1 - KCa_1 + 3KCa_1a_2' - KCa_2 + 3Ka_2a_1'}{-3KCa_2 - 3Ka_1 - 3Ca_2' + 9a_2'a_1 + 3KCa_2' + 3Ka_1' + 3Ca_2 - 9a_1'a_2}$$

(1)

Here in (1), a_1, a_2, a_1', and a_2' are the weights that determine the two target points. The fraction must be factored so that the weights can be divided out and thus eliminated. K and C are two constants that depend solely on the basepoints.

Week of April 30, 2008

This past week has seen a little progress with finding the intersect point of all the lines from the plotted sequences. Since the previous factoring prob-

lem seemed so daunting, I tried another approach at finding the intersect point. Basically, since I believe the intersect point to be at the center of the triangle formed by the basepoints, I set out to manually plot the basepoints so that I could legitimately find the center of the triangle, and then plug that center point into the equation of the lines, and see if that point would be on every line. Unfortunately, I had a problem converting the basepoints into xy-coordinates. It seems that my conversion of the second basepoint was off, and a dropped negative appeared to be the culprit (running the program and making it print certain values seemed to confirm this). I then went back to the actual code that performs this operation, and discovered that the code had a different inner-product than what the fall-quarter report (which I was working from) had. So I retried with the correct inner product and got the right answer, sparing myself any doubt I may have developed in all of my work-by-hand thus far. After legitimately finding the center point of the triangle, I found myself with another difficult factoring problem. I then discovered an error in my formula for the equation of the lines, and fixed it. So now with a little more work, I should have the intersection proved shortly.

I found that the line for an approximation is plotted perpendicular to the plane, so that we only see a black dot, whenever any of the weights on the basepoints for the target point is equal to zero.

I have also been thinking of exactly what to put on my poster for the research expo, and trying to make some sense out of the template sent to me by Dr. Mohlenkamp. In the next week, I plan on resolving this intersection issue, and making tremendous progress on the poster, since the expo is only a couple weeks away now.

Week of May 14, 2008

So I forgot to write a journal last week. To recap, two weeks ago I was having trouble showing that the intersection of all the sequence lines was the center point of the triangle of basepoints. So, I started over completely, re-did all the work from this entire quarter, found about two mistakes I had made previously, and achieved the intended proof. Thus, I finished my research. I spent this past week working on my poster for the Research and Creative Activity Expo. After several revisions and some LaTeX frustra-

tions, I finally got the poster made, and it is ready to go.

Research Expo and the Remainder of Spring

In making my poster, I learned a couple of things. First, it is not a good idea to procrastinate. I did not procrastinate, but it became clear to me that, if I had, I would have had a real problem meeting my deadline, which was the date of the expo. This fact became clear to me after I had also learned that tailoring a poster or a presentation to a specific audience, especially one that is not knowledgeable about your subject matter, takes a lot of time, and a lot of revision.

The Research and Creative Activity Expo was a new and good experience. I had the opportunity to present my poster to an audience that would not necessarily have any foreknowledge of the subject of the research. I presented my research to two judges the night before the expo was officially opened. They seemed to be at least a little bit interested, and I believe that they were not entirely confused. Overall, I believe the presentation went well, and I am glad for the opportunity to present mathematical work to an untrained audience.

For the remainder of the quarter, I had no new research to conduct, and I already had a complete poster for the poster session that Dr. Mohlenkamp had hosted. This poster session was catered to an audience that primarily had a mathematical background, since it was advertised and held in Morton Hall. Since my poster was made specifically for the expo, it was simpler than it could have been for the mathematical poster session. From this I learned that, once a presentation, poster, or paper is written and tailored to a specific audience, it is likely only useful once.