MODELING AURORAS

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Outline

INTRODUCTION

Introduction

- Modeling natural phenomena is an important research subject in many areas
- NASA collects satellite images to study forms of auroras
- Auroras are related to *plasma* the fourth state of matter
- Baronoski and others developed the first model to simulate auroras based on physics

Northern Lights

OVERVIEW OF AURORAL PHYSICS

Why can the Lights only be seen in some places?

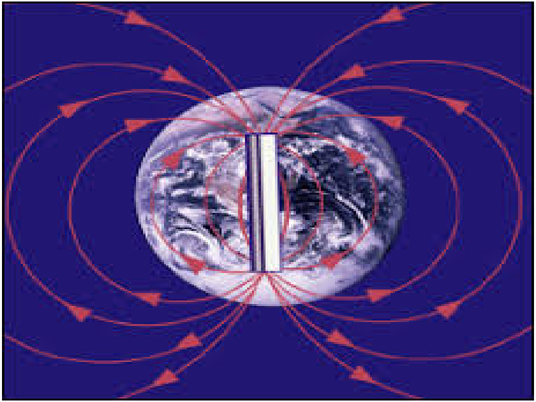


Figure: Solar winds are drawn towards the poles

Mathematics Governing Plasma

Charged particles in plasma obey the following:

- The electric and magnetic fields $E(x(t), t)$ and $B(x(t), t)$ can be found
- Charged particles obey the Lagrangian Equation

$$L(x, v, t) = \frac{1}{2}m|v|^2 - q\varphi(x) + qv \cdot A(x)$$

- The rates of change are given by the Euler-Lagrange Equations

$$\frac{d}{dt}\frac{\partial L}{\partial v_i} = \frac{\partial L}{\partial x_i}$$

- These equations are equivalent to the Lorentz force law:

$$F = q(E + v \times B)$$

Mathematics of Electromagnetic Fields

Particles are influenced by magnetic fields so they obey Maxwell's equations.

1. $\nabla \cdot E = \frac{\rho}{\epsilon_0}$ where ρ is the charge density and ϵ_0 is the permissivity of free space (a constant)
2. $\nabla \times E + \frac{1}{q}\frac{\partial B}{\partial t} = 0$
3. $\nabla \cdot B = 0$ and
4. $\nabla \times B - \frac{1}{q}\frac{\partial E}{\partial t} = \frac{4\pi}{q} J$ where J is the current

METHODS TO SOLVE ELECTROMAGNETIC EQUATIONS

Analytic Methods

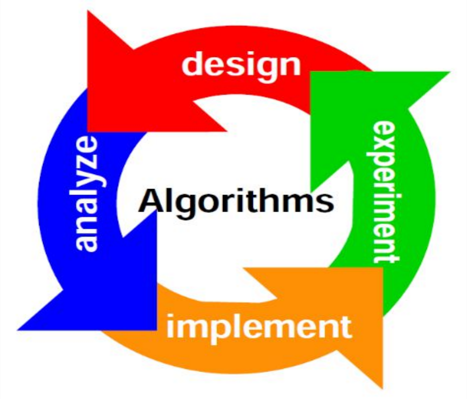
Solving these equations analytically is impossible in most situations. We come up with approximations based on kinetic theory.

- Use kinetic theory assumptions to derive piece-wise distribution functions f_i
- Smooth them using an averaging procedure and create a collision operator $C(f)$
- The Boltzmann collision operator includes random binary collisions

Why use numerical methods?

A computer is suitable for finding numerical approximations for many types of problems that cannot be solved analytically

- Simple calculations that are done iteratively
- Linear algebra in higher dimensions
- Updating random variables in real time



Numerical Methods

Numerical methods give approximate solutions in order to model auroras.

$$\nabla \cdot E = \nabla^2 \phi = \frac{\rho}{\epsilon_0}$$

1. Discretize the given function ρ
2. Approximate a solution using finite difference (FD) method
3. Use a multigrid method to come up with a reasonable approximation

Multigrid Method Example

Using Taylor expansion and FD method we have an approximation

$$\frac{\partial^2 \phi}{\partial x^2}(x) \approx \frac{\phi(x+h) - 2\phi(x) + \phi(x-h)}{h^2}.$$

Solve a linear system of equations and find an approximate solution.

$$A\phi(x_i) = \rho_i$$

The residuals on the fine mesh grid

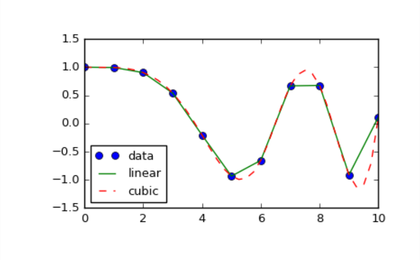
$$R^F = \rho - A^F \phi^F(x_i)$$

Multigrid Method

Transfer the residuals to a coarse grid and solve for ϕ^C.

$$R^{C \leftarrow F} = A^C \phi^C$$

Obtain a fine mesh correction using interpolation $\phi^{F \leftarrow C}$

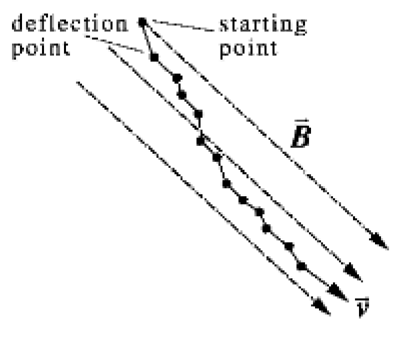


Update fine mesh solution $\phi^F_{new} = \phi^F_{old} + \phi^{F \leftarrow C}$

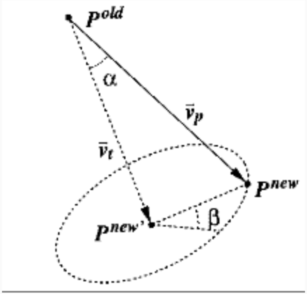
MODEL STARTING POINTS

How Light is Emitted

- Electrons are highly energized particles entering Earth's atmosphere.
- Collide with other particles in the atmosphere and may emit light
- A typical electron bean can have 300 separate collisions



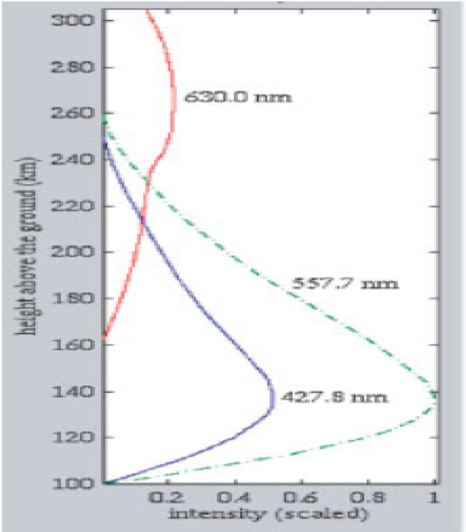
Stochastic Nature of Electron Beams



- α is the pitch angle and is randomly perturbed
- β perturbs the coordinates of the new position once the vertical displacement occurs

Colors of Lights

- The model uses red, green, and blue colors
- Each color has an average intensity based on the height off the ground
- Other wavelengths of light have different intensity distributions



Forms of Auroras

Many typical forms are present in natural auroras. The model focuses on the two most common forms:

1. Curtains
2. Rayed bands

Both types of auroral forms have sinusoidal boundaries.

Later models implement other phenomena including spirals

Model Parameters

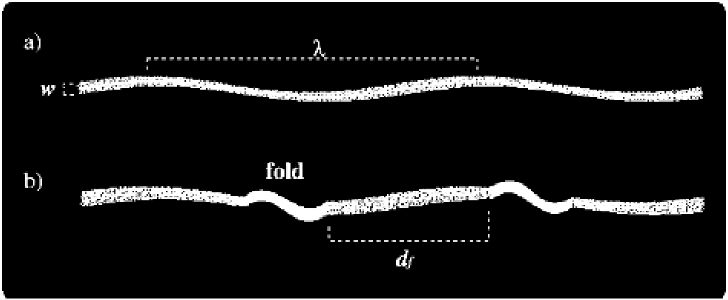


Figure: Top-down view of a sheet in auroral display

- n is the number of sampling points in each sheet
- w is the sheet width λ is the wavelength
- d_f is the distance between folds Φ_s is the phase transition

Intensity of Lights

Intensity of light is based on many things:

1. Height from the ground
2. Auroral Form
3. Pitch angle α and distance from magnetic lines
4. Distance between collisions d_t and velocity v_t

d_t is randomly perturbed which matches actual phenomenon

Blending of Lights

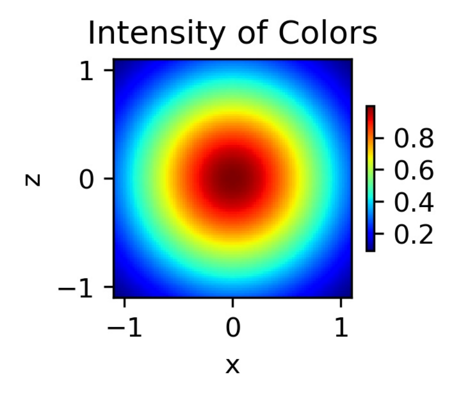
At each point of the electron beam trajectory we put a grid to determine the emission of light in the following way:

1. Using α, β, v_t and d_t find the position of your new point. It will be in some square of the grid.
2. Based upon height blend the red, green, and blue outputs based upon the intensity at that point

$$\begin{bmatrix} \\ R \\ G \\ B \\ \\ \end{bmatrix} = \begin{bmatrix} \\ Std \\ Conv \\ Matrix \\ \\ \end{bmatrix} \begin{bmatrix} \\ X \\ Y \\ Z \\ \\ \end{bmatrix}$$

Adjusting Light Output

We adjust intensity using a Gaussian distribution



WORKING THE MODEL

Resolving the Model

Mathematically, using the gridlines a fine mesh can be created in order to numerically resolve the Maxwell Equations and the Lorentz force equation at each iteration.

1. $\nabla \cdot E = \frac{\rho}{\epsilon_0}$=$\nabla^2\phi$ the Laplacian of electrostatic potential.
2. Discretize both sides to make a linear system $A\phi = \rho_i$.
3. Solve the linear system using numerical methods (FFT or Gauss-Seidel)
4. Implement a multigrid method to find ϕ

Resolving the Model

$$E = \nabla\phi, \qquad \nabla \times E + \frac{1}{q}\frac{\partial B}{\partial t} = 0$$

Once we have an approximation of ϕ

1. Use ϕ to find the electric field E
2. Use E to find B and resolve grid position and velocity
3. Move on to next iteration and repeat

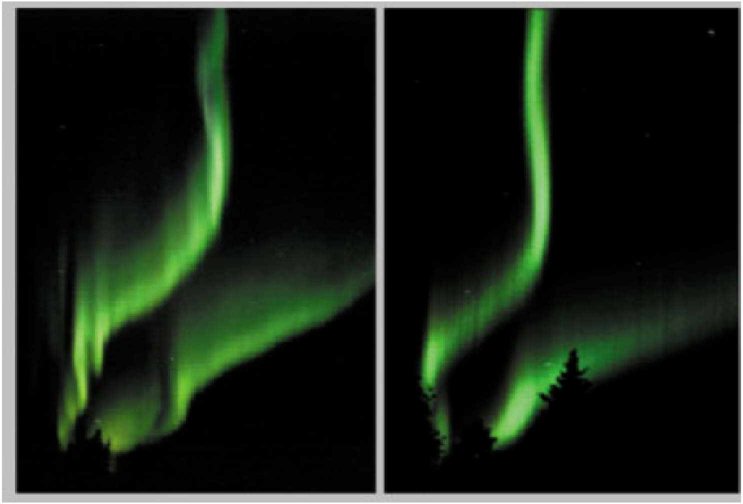


Figure 19. Left: Photograph of two ray-filled curtains (courtesy of Jan Curtis, 5 s of exposure time). Right: Simulation of auroral curtain showing the ray structure.

CONCLUSION

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The purpose of the research is to model natural occurences of plasma phenomena

The model uses many physical and mathematical tools

- The electro-magnetic field equations
- Numerical methods to solve partial differential equations behind the Northern Lights
- Stochastic nature of the particles is found in other plasma physics

Thank You!