score	possible	page
	20	1
	30	2
	30	3
	20	4
	100	

Name: ______________________________

Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student's solutions. (You may ask me questions.)

1. Suppose f and g are differentiable functions with the following properties:

$$\begin{array}{lll} f(0)=2 & f(1)=0 & f(2)=1 \\ g(0)=1 & g(1)=2 & g(2)=0 \\ \int_0^1 f(x)\,dx=\pi & \int_1^2 f(x)\,dx=\pi^3 & \int_2^3 f(x)\,dx=\pi^5 \\ \int_0^1 g(x)\,dx=\sqrt{2} & \int_1^2 g(x)\,dx=\sqrt{3} & \int_2^3 g(x)\,dx=\sqrt{5} \\ f'(0)=e & f'(1)=e^3 & f'(2)=e^5 \\ g'(0)=\sqrt{7} & g'(1)=\sqrt{11} & g'(2)=\sqrt{13} \end{array}$$

Evaluate the following. If one cannot be evaluated with the given information, write "NOT ENOUGH INFORMATION." You do **not** need to justify your answer or show your work.

/2 (a) $\int_0^3 g(x)\,dx = \int_0^1 g(x)\,dx + \int_1^2 g(x)\,dx + \int_2^3 g(x)\,dx = \sqrt{2}+\sqrt{3}+\sqrt{5}$

/2 (b) $\int_3^2 f(x)\,dx = -\int_2^3 f(x)\,dx = -\pi^5$

/2 (c) $\int_1^2 (5g(x)+f(x))\,dx = 5\int_1^2 g(x)\,dx + \int_1^2 f(x)\,dx = 5\sqrt{3}+\pi^3$

/2 (d) $\int_0^2 \frac{f(x)}{g(x)}\,dx$ Not enough information.

/2 (e) $\left(\frac{f}{g}\right)'(0) = \frac{f'(0)g(0)-f(0)g'(0)}{(g(0))^2} = \frac{e\cdot 1 - 2\sqrt{7}}{1^2}$

/2 (f) $\int_0^9 f(x)\,dx - \int_2^9 f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx = \pi+\pi^3$

/2 (g) $\int_0^2 g'(r)\,dr = g(2)-g(0) = 0-1$

/2 (h) $\int_7^7 g''(x)\,dx = 0$

/2 (i) $\lim_{x\to 0} \frac{f(x)-1}{g(x)} = \frac{2-1}{1} = 1$

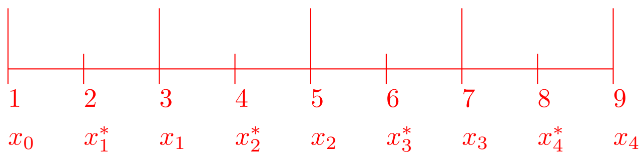
/2 (j) $\lim_{h\to 0} \frac{g(1+h)-2}{h} = g'(1) = \sqrt{11}$

/10 2. Use the Midpoint rule with $n = 4$ to approximate the integral. (Do **not** simplify.) Include a drawing of your subdivision of the interval and the midpoints used in the approximation.

$$\int_1^9 \sin(\sqrt{x})dx$$

[Similar to 5.2#11] The interval $[a, b] = [1, 9]$ has length 8 and we are using 4 rectangles, so the width of each rectangle is $\Delta x = 2$. The base of the first rectangle is $[1, 3]$, which has midpoint $x_1^* = 2$. The second has base $[3, 5]$ and midpoint $x_2^* = 4$, the third has base $[5, 7]$ and midpoint $x_3^* = 4$, and the fourth has base $[7, 9]$ and midpoint $x_4^* = 8$. The area estimate is thus

$$f(x_1^*)\Delta x + f(x_2^*)\Delta x + f(x_3^*)\Delta x + f(x_4^*)\Delta x = \sin(\sqrt{2}) \cdot 2 + \sin(\sqrt{4}) \cdot 2 + \sin(\sqrt{6}) \cdot 2 + \sin(\sqrt{8}) \cdot 2\,.$$



/10 3. The velocity of a fox increased steadily during the first three seconds it tried to catch a rabbit. Its velocity at half second intervals is given in the table. Find good upper and lower estimates for the distance that it traveled during these three seconds. Do **not** simplify.

t (s)	0	.5	1	1.5	2	2.5	3
v (ft/s)	0	5.2	10.8	14.9	18.1	19.6	20.2

Since its velocity is increasing, using the left edge of each time interval will give a lower estimate and using the right edge will give an upper estimate. The lower estimate is

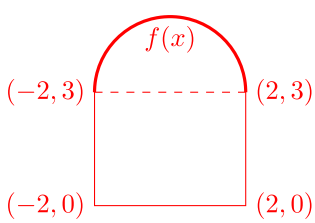
$$\frac{1}{2}(0 + 5.2 + 10.8 + 14.9 + 18.1 + 19.6) \text{ ft}$$

and the upper estimate is

$$\frac{1}{2}(5.2 + 10.8 + 14.9 + 18.1 + 19.6 + 20.2) \text{ ft}\,.$$

/10 4. Sketch the function $f(x) = 3 + \sqrt{4 - x^2}$ on the interval $[-2, 2]$.

Then evaluate the integral $\displaystyle\int_{-2}^{2} f(x)\,dx$ by interpreting it in terms of areas.



[Similar to 5.2#29–36]

The integral is the area of a rectangle plus the area of half a disc. The rectangle has width 4 and height 3 and so area 12. The disc has radius 2 and so the half disc has area $\frac{1}{2}\pi 2^2 = 2\pi$. Thus $\displaystyle\int_{-2}^{2} f(x)\,dx = 12 + 2\pi$.

5. Evaluate the integrals. Do **not** simplify the result.

/6 (a) $\int_{-3}^{2} (x^2 - 5)\, dx =$ [Similar to 5.3#1]

$$= x^3/3 - 5x\Big]_{-3}^{2} = (2^3/3 - 5 \cdot 2) - ((-3)^3/3 - 5(-3)) .$$

/6 (b) $\int_{1}^{\pi} \frac{x^4 + 3}{x}\, dx =$ [Similar to 5.3#23]

$$= \int_{1}^{\pi} x^3 + \frac{3}{x}\, dx = x^4/4 + 3\ln(|x|)\Big]_{1}^{\pi} = (\pi^4/4 + 3\ln(\pi)) - (1^4/4 + 3\ln(1)) .$$

/6 (c) $\int (x+2)(3\sqrt{x} + 5)\, dx =$ [Similar to 5.3#5 and 11]

$$= \int 3x^{3/2} + 5x + 6x^{1/2} + 10\, dx = 3\frac{x^{5/2}}{5/2} + 5\frac{x^2}{2} + 6\frac{x^{3/2}}{3/2} + 10x + C .$$

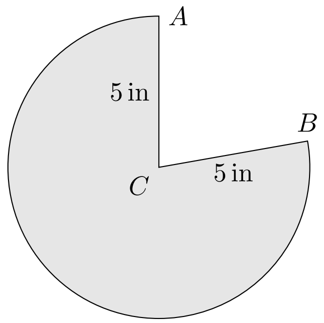
/6 (d) $\int_{0}^{1/2} \frac{5}{\sqrt{1-t^2}}\, dt =$ [Similar to 5.3#21]

$$= 5\arcsin(t)\Big]_{0}^{1/2} = 5\arcsin(1/2) - 5\arcsin(0) .$$

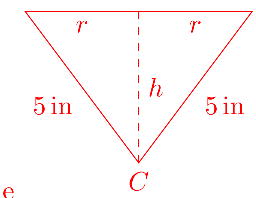
/6 (e) $\int_{2}^{3} 10^{x+7}\, dx =$ [Similar to 5.3#15 and 25]

$$= 10^7 \frac{10^x}{\ln(10)}\Big]_{2}^{3} = 10^7\frac{10^3}{\ln(10)} - 10^7\frac{10^2}{\ln(10)} .$$

/20 6. A cone-shaped drinking cup is made from a circular piece of paper of radius $5\,\text{in}$ by cutting out a sector and joining the edges CA and CB. Find the maximum capacity of such a cup.



[Similar to 4.5#29] A side view of the cone-shaped cup is the triangle



The volume of the cone is $V = \frac{1}{3}\pi r^2 h$. The pythagorean theorem gives the constraint $(5\,\text{in})^2 = r^2 + h^2$. Solving for r^2 gives $r^2 = (5\,\text{in})^2 - h^2$ and substituting into V gives

$$V = \frac{1}{3}\pi((5\,\text{in})^2 - h^2)h\,.$$

Differentiating with respect to h gives

$$V' = \frac{1}{3}\pi((5\,\text{in})^2 - 3h^2)$$

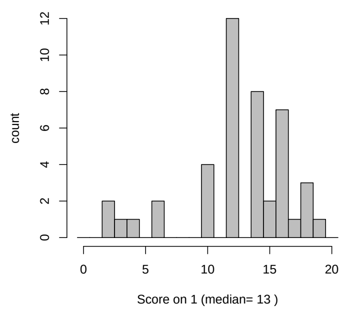
and differentiating again gives

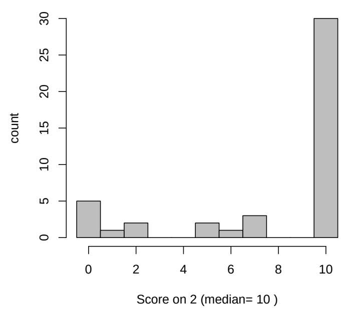
$$V'' = \frac{1}{3}\pi(-6h)\,.$$

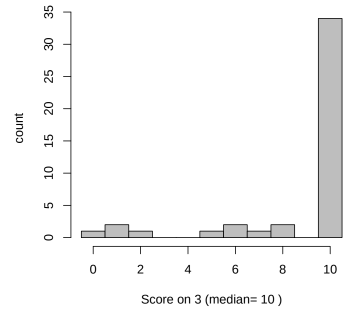
Setting $V' = 0$ yields $h = \frac{5}{\sqrt{3}}\,\text{in}$ as the only positive critical number. Since $V'' < 0$ for $h > 0$, this critical number gives a maximum. Inserting into V gives the maximum volume

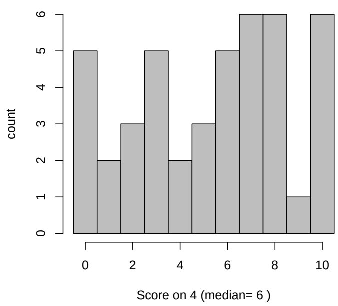
$$V = \frac{1}{3}\pi\left(5^2 - \frac{5^2}{3}\right)\frac{5}{\sqrt{3}}\,\text{in}^3 = \frac{2\cdot 5^3\pi}{3^2\sqrt{3}}\,\text{in}^3\,.$$

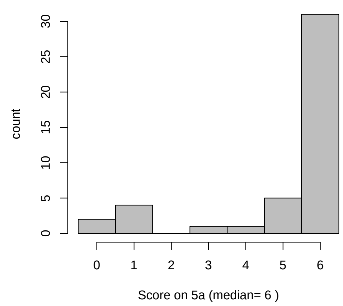
Scores

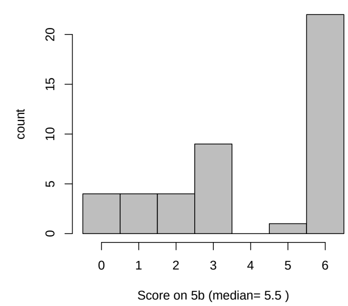

count
Score on 1 (median= 13)

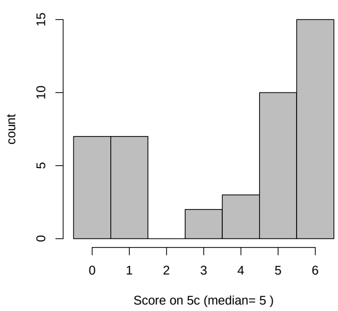

count
Score on 2 (median= 10)

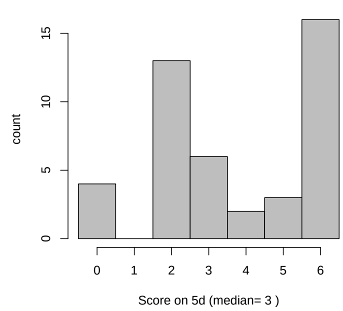

count
Score on 3 (median= 10)

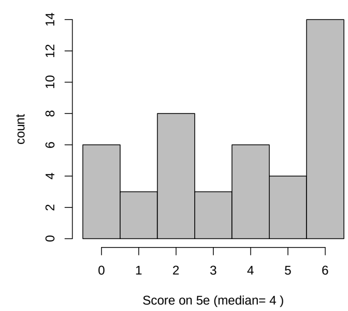

count
Score on 4 (median= 6)

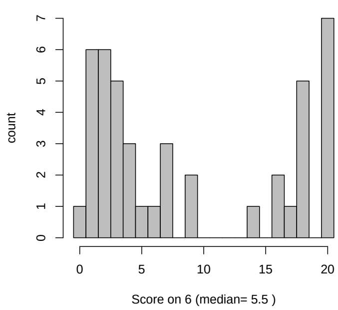

count
Score on 5a (median= 6)

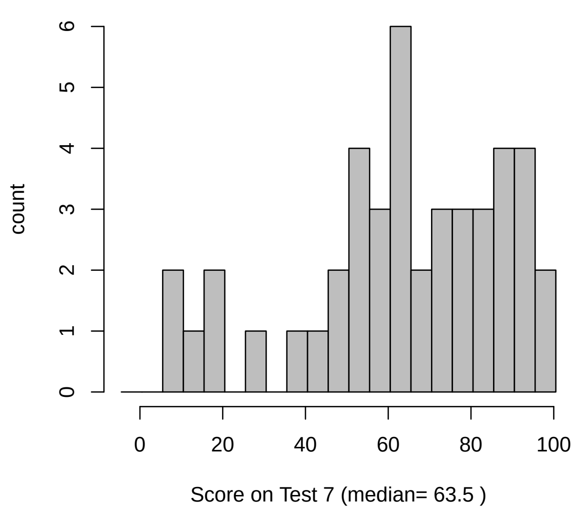

count
Score on 5b (median= 5.5)


count
Score on 5c (median= 5)


count
Score on 5d (median= 3)


count
Score on 5e (median= 4)


count
Score on 6 (median= 5.5)


count
Score on Test 7 (median= 63.5)