score	possible	page
	30	1
	25	2
	25	3
	20	4
	100	

Name: ______________________________

Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student's solutions. (You may ask me questions.)

1. Determine whether each of the following statements is True or False.
Correct answers are worth +3, incorrect answers are worth −2, and no answer is worth +1.

/3 (a) True False If $x > 0$ then $\dfrac{\ln(x+2)}{\ln(x)} = \ln(2)$.
False. There are no simplifications for division of logarithms.

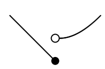
/3 (b) True False If $x \neq 0$ then $\left(\dfrac{25}{4x^4}\right)\left(\dfrac{5}{x^3}\right)^{-3} = \dfrac{2x^5}{5}$.
False. $\left(\dfrac{25}{4x^4}\right)\left(\dfrac{5}{x^3}\right)^{-3} = \left(\dfrac{25}{4x^4}\right)\left(\dfrac{x^3}{5}\right)^{3} = \left(\dfrac{25}{4x^4}\right)\left(\dfrac{x^9}{5^3}\right) = \dfrac{25x^9}{4x^4 \cdot 5^3} = \dfrac{x^5}{20} \neq \dfrac{2x^5}{5}$.

/3 (c) True False If $x \neq 0$ then $x + x^{-1} = 0$.
False. If $x = 1$ then $x + x^{-1} = 2$.

/3 (d) True False If $\sin(t) \neq 0$ then $\sin(t)\cot(t) = \cos(t)$.
True. $\cot(t) = \frac{\cos(t)}{\sin(t)}$

/3 (e) True False If $\lim_{x\to a} f(x) = f(a)$ then f is continuous at a.
True. That is the definition of continuous at a point.

/3 (f) True False If $\lim_{x\to a} f(x) = f(a)$ then f is differentiable at a.
False. That is the definition of continuous at a point, which is necessary but not sufficient to make f differentiable.

/3 (g) True False The graph  shows a jump discontinuity.
True. The limits from each side exist but disagree, which is the definition of a jump discontinuity.

/3 (h) True False If f and g are differentiable, then $\dfrac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$.
True. This is a basic differentiation formula, which the book calls the difference rule.

/3 (i) True False A function can have two different horizontal asymptotes.
True. It can have one asymptote as $x \to -\infty$ and another as $x \to \infty$.

/3 (j) True False If $f(x) \leq h(x)$, $g(x) \leq f(x)$, $h(x) \leq 0$, $\lim_{x\to a} g(x) = K$, and $\lim_{x\to a} h(x) = K$, then $\lim_{x\to a} f(x) = K$.
True. This is an awkwardly written version of the Squeeze Theorem. The assumption $h(x) \leq 0$ is not needed.

2. Compute the following limits. **Do not use L'Hôpital's rule!**

/5 (a) $\lim_{x \to \pi^+} \csc(x) =$

$\lim_{x \to \pi^+} \frac{1}{\sin(x)} = \lim_{t \to 0^-} \frac{1}{t} = -\infty$. We used knowledge of the graph of $\sin(x)$, that $\sin(\pi) = 0$ and $\sin(x) < 0$ for x slightly bigger than π.

/5 (b) $\lim_{x \to 4^+} \frac{x+3}{x-4} =$

$$\lim_{x \to 4^+} \frac{7}{x-4} = \lim_{t \to 0^+} \frac{7}{t} = \infty\,.$$

/5 (c) $\lim_{x \to -\infty} \frac{5x^3 + 9x^5 + 2}{11x^5 - 13} =$

Multiplying the numerator and denominator by x^{-5} yields

$$\lim_{x \to -\infty} \frac{5x^{-2} + 9 + 2x^{-5}}{11 - 13x^{-5}} = \frac{9}{11}\,.$$

/10 3. Using **the definition of the derivative as a limit**, compute the derivative of $f(x) = (3x-1)^{-1}$.

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} && = \lim_{h \to 0} \frac{(3(x+h)-1)^{-1} - (3x-1)^{-1}}{h} \\ &= \lim_{h \to 0} \frac{\frac{1}{(3(x+h)-1)} - \frac{1}{(3x-1)}}{h} && = \lim_{h \to 0} \frac{\frac{(3x-1)-(3(x+h)-1)}{(3x-1)(3(x+h)-1)}}{h} \\ &= \lim_{h \to 0} \frac{-3h}{h(3x-1)(3(x+h)-1)} && = \lim_{h \to 0} \frac{-3}{(3x-1)(3(x+h)-1)} \\ &= \frac{-3}{(3x-1)(3(x+0)-1)} && = \frac{-3}{(3x-1)^2}\,. \end{aligned}$$

/5 4. Given that $f(3) = 5$ and $f'(3) = 7$, write an equation for the tangent line to the graph $y = f(x)$ at $x = 3$.

Using point-slope form, the tangent line is $y - f(a) = f'(a)(x - a)$. Plugging in the given values yields $y - 5 = 7(x - 3)$.

5. Compute the following derivatives. (You can use derivative rules, rather than computing the limits.)

/2 (a) $f(x) = x^2 \Rightarrow f'(x) =$ $2x$

/2 (b) $f(x) = \dfrac{1}{x^2} \Rightarrow f'(x) =$ $-2x^{-3}$

/2 (c) $f(x) = \sqrt{x} \Rightarrow f'(x) =$ $\frac{1}{2}x^{-1/2}$

/2 (d) $f(x) = \dfrac{1}{\sqrt{x}} \Rightarrow f'(x) =$ $-\frac{1}{2}x^{-3/2}$

/2 (e) $f(x) = x^{3/4} \Rightarrow f'(x) =$ $\frac{3}{4}x^{-1/4}$

/2 (f) $f(x) = \sin(x) \Rightarrow f'(x) =$ $\cos(x)$

/2 (g) $f(x) = \sin(7) \Rightarrow f'(x) =$ 0

/2 (h) $f(x) = x^{-7} \Rightarrow f'(x) =$ $-7x^{-8}$

/2 (i) $f(x) = x^2 + 5 \Rightarrow f'(x) =$ $2x$

/2 (j) $f(x) = 5x^2 \Rightarrow f'(x) =$ $10x$

/5 6. (a) State the Intermediate Value Theorem using the template below.

If
- f is continuous on $[a, b]$ and
- $f(a) < N < f(b)$ or $f(a) > N > f(b)$,

then

there exists $c \in (a, b)$ such that $f(c) = N$.

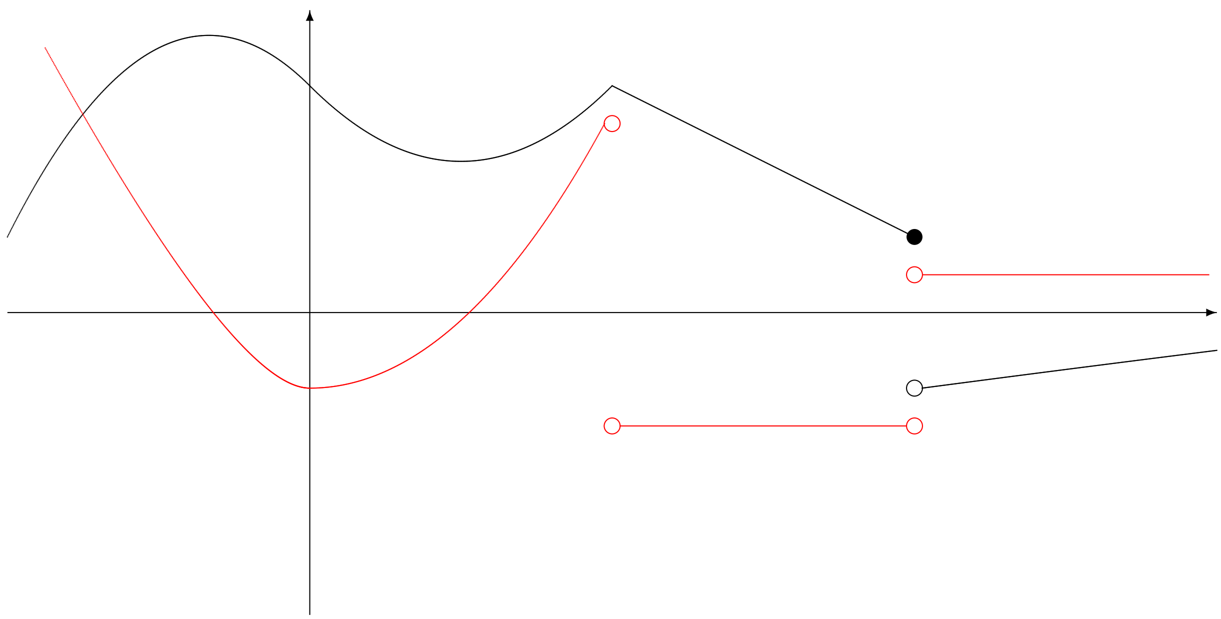
/5 (b) Use the Intermediate Value Theorem to show that the equation $\cos(x) = 2x^2$ has a solution.

Let $f(x) = \cos(x) - 2x^2$, so we want to show a solution to $f(x) = 0$ exists. Since $\cos(x)$ and x^2 are both continuous, so is $f(x)$. Plugging in, we find

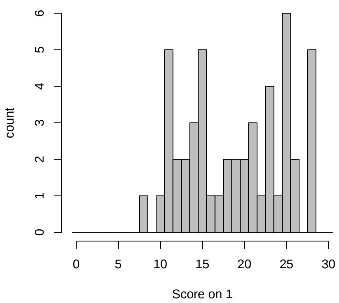
$$f(0) = 1 - 0 = 1 > 0 \quad \text{and}$$
$$f(\pi/2) = 0 - 2(\pi/2)^2 < 0\,.$$

So, by the Intermediate Value Theorem, there must exist $0 < c < \pi/2$ such that $f(c) = 0$.

/10 7. The graph of a function f is given below. On the same axes, sketch the graph of f'.



Scores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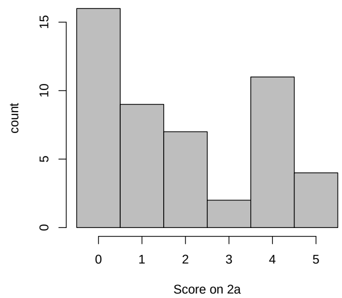
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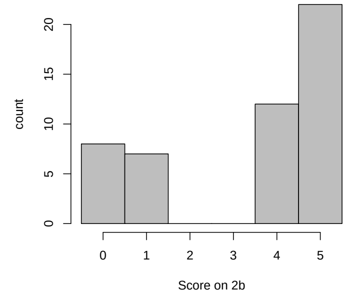
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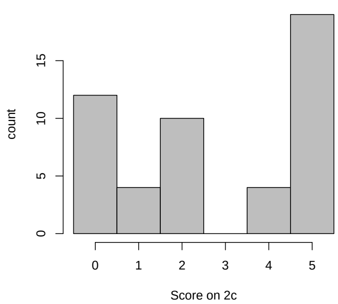
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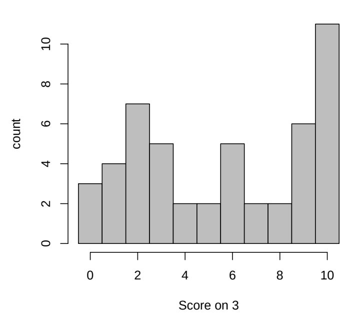
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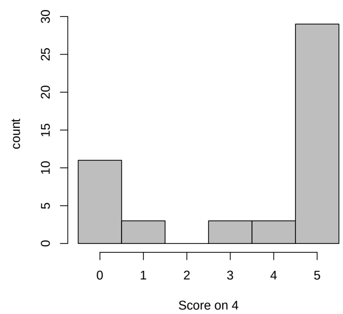
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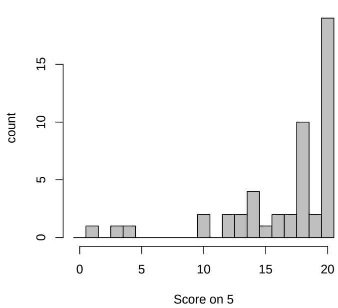
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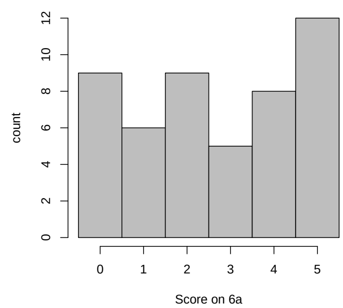
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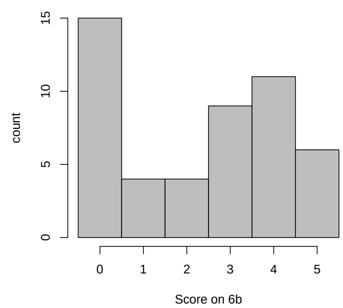
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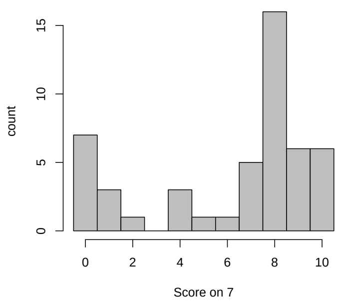
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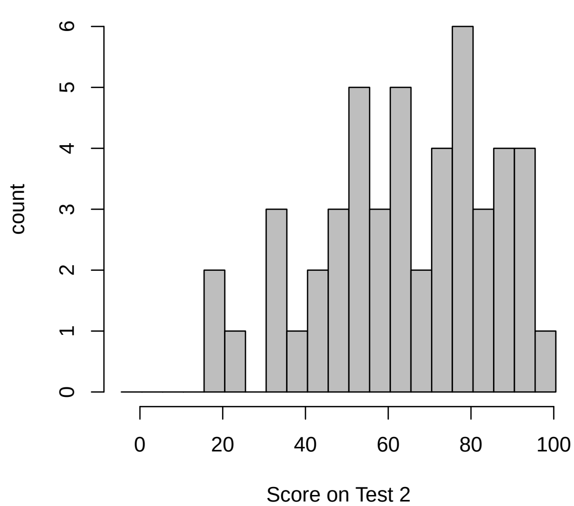
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Score on Test 2