score	possible	page
	30	1
	30	2
	20	3
	20	4
	100	

Name: ______________________________

Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student's solutions. (You may ask me questions.)

1. Determine whether each of the following statements is True or False.
   Correct answers are worth +3, incorrect answers are worth −2, and no answer is worth +1.

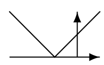
/3 (a) True False If $f(x) = (3x-1)^{-1}$ then $f(x+h) = \frac{1}{3x-1+h}$.
False. $f(x+h) = \frac{1}{3(x+h)-1} = \frac{1}{3x-1+3h}$

/3 (b) True False If $x \neq 0$, $y \neq 0$, and $z \neq 0$, then $\dfrac{\frac{x}{y}}{z} = \dfrac{x}{yz}$.
True. Dividing by y and then z is the same as dividing by yz.

/3 (c) True False $5 + \cos^2(\theta) + \sin^2(\theta) = 6$.
True. $\sin^2(\theta) + \cos^2(\theta) = 1$

/3 (d) True False If $x > 1$ then $7^{\log_7(x)} = x$.
True. Exponentials and logarithms are inverse functions. $x > 1$ is sufficient to stay in the domain of the logarithm.

/3 (e) True False If $x > 0$ then $\dfrac{\ln(x+2)}{\ln(x)} = \ln(2)$.
False. There are no simplifications for division of logarithms.

/3 (f) True False The graph of $f(x) = |x-1|$ looks like 
False. The argument $x - 1$ shifts the graph of $|x|$ to the right.

/3 (g) True False If $\lim_{x\to 3} f(x) = 4$ then $\lim_{x\to 3^+} f(x) = 4$.
True. For the ordinary limit to exist, both one-sided limits must exist and agree with it.

/3 (h) True False If $\lim_{x\to 3} f(x) = 4$ then $f(3) \neq 4$.
False. The limit gives no information about the value of $f(3)$, so it could be 4.

/3 (i) True False If $x \neq 0$ then $\left(\dfrac{25}{4x^4}\right)\left(\dfrac{5}{2x^3}\right)^{-3} = \dfrac{2x^5}{5}$.

True. $\left(\dfrac{25}{4x^4}\right)\left(\dfrac{5}{2x^3}\right)^{-3} = \left(\dfrac{25}{4x^4}\right)\left(\dfrac{2x^3}{5}\right)^{3} = \left(\dfrac{25}{4x^4}\right)\left(\dfrac{2^3x^9}{5^3}\right) = \dfrac{25\cdot 2^3x^9}{4x^4\cdot 5^3} = \dfrac{2x^5}{5}$.

/3 (j) True False The equation $y-3 = 4(x-5)$ describes a line with slope 3 through the point $(4, 5)$.

False, It is a line with slope 4 through the point $(5, 3)$.

2. Compute the following limits. **Do not use L'Hôpital's rule!**

/10 (a) $\lim_{x\to 2} \frac{x-2}{x^2+x-6} =$

$$\lim_{x\to 2} \frac{x-2}{(x-2)(x+3)} = \lim_{x\to 2} \frac{1}{x+3} = \frac{1}{2+3} = \frac{1}{5}.$$

/10 (b) $\lim_{h\to 0} \frac{h}{(2+h)^3-8} =$

$$\lim_{h\to 0} \frac{h}{(2^3+3(2^2h)+3(2h^2)+h^3)-8} = \lim_{h\to 0} \frac{h}{12h+6h^2+h^3} = \lim_{h\to 0} \frac{h}{h(12+6h+h^2)}$$
$$= \lim_{h\to 0} \frac{1}{12+6h+h^2} = \frac{1}{12+0+0} = \frac{1}{12}.$$

/10 (c) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} =$

$$\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}\frac{\sqrt{x}+3}{\sqrt{x}+3} = \lim_{x\to 9} \frac{x-9}{(x-9)(\sqrt{x}+3)} = \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}.$$

3. Compute the following limits. **Do not use L'Hôpital's rule!**

/10 (a) $\lim_{x\to -4} \dfrac{x^{-1}+4^{-1}}{x+4} =$

$$\lim_{x\to -4} \frac{\frac{1}{x}+\frac{1}{4}}{x+4} = \lim_{x\to -4} \frac{\frac{4}{4x}+\frac{x}{4x}}{x+4} = \lim_{x\to -4} \frac{\frac{4+x}{4x}}{x+4} = \lim_{x\to -4} \frac{4+x}{4x(x+4)} = \lim_{x\to -4} \frac{1}{4x} = \frac{1}{4(-4)} = \frac{1}{-16}\,.$$

/10 (b) $\lim_{\theta\to 0} \dfrac{\tan(5\theta)}{\sin(3\theta)} =$

$$\lim_{\theta\to 0} \frac{\frac{\sin(5\theta)}{\cos(5\theta)}}{\sin(3\theta)} = \lim_{\theta\to 0} \frac{\sin(5\theta)}{\cos(5\theta)\sin(3\theta)} = \lim_{\theta\to 0} \frac{1}{\cos(5\theta)}\frac{5}{3}\frac{\sin(5\theta)}{5\theta}\frac{3\theta}{\sin(3\theta)} = \frac{1}{1}\frac{5}{3}(1)(1) = \frac{5}{3}\,.$$

/5 4. (a) State the Squeeze Theorem using the template below.

If
- $f(x) \le g(x) \le h(x)$ when x is near a (except possibly at a) and
- $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$,

then $\lim_{x\to a} g(x) = L$.

/5 (b) Use the Squeeze Theorem to evaluate $\lim_{x\to 0} x \sin\left(\frac{1}{x}\right)$.

Set

$$f(x) = -|x|,$$
$$g(x) = x\sin\left(\frac{1}{x}\right), \quad \text{and}$$
$$h(x) = |x|.$$

Since $-1 \le \sin(\cdot) \le 1$, we have $f(x) \le g(x) \le h(x)$ when x is near 0 (and for all $x \ne 0$). We can compute $\lim_{x\to 0} f(x) = \lim_{x\to 0} h(x) = 0$, so the assumptions of the Squeeze Theorem are satisfied and we can conclude $\lim_{x\to a} g(x) = 0$.

5. Sketch the graph of a single function f that has all of the following properties:

/2 (a) f has domain $[-3, 3]$.

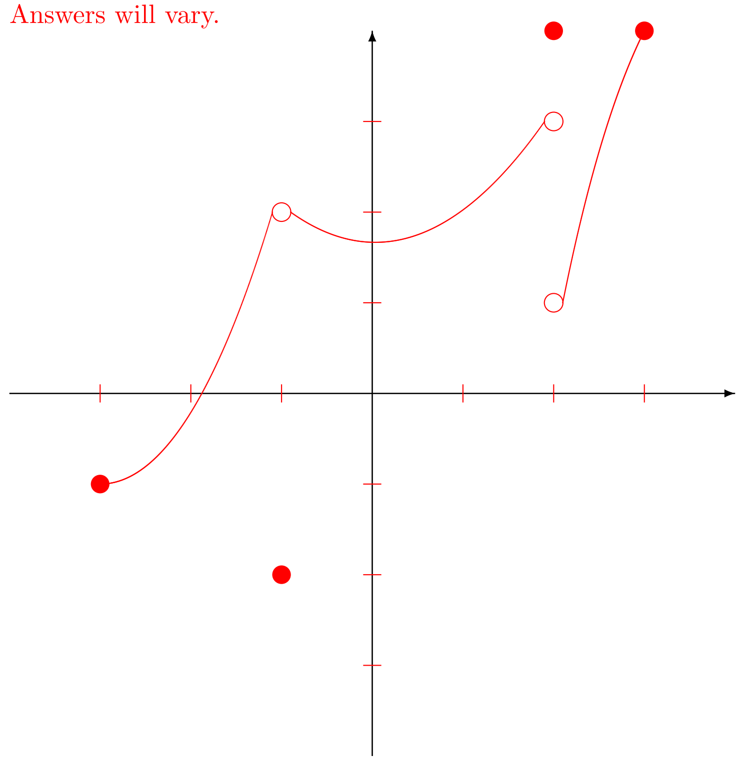
/2 (b) $\lim_{x\to 2^+} f(x) = 1$.

/2 (c) $\lim_{x\to 2^-} f(x) = 3$.

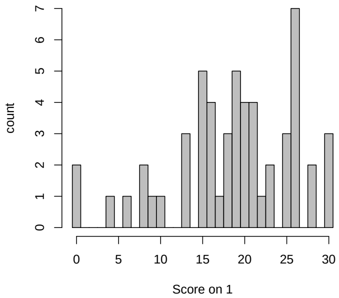
/2 (d) $\lim_{x\to -1} f(x) = 2$.

/2 (e) $f(-1) = -2$.

Answers will vary.



Scores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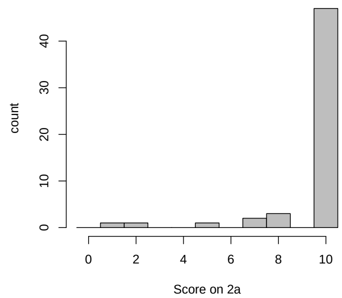
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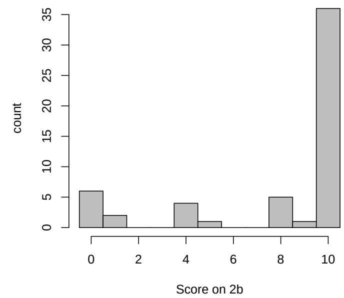
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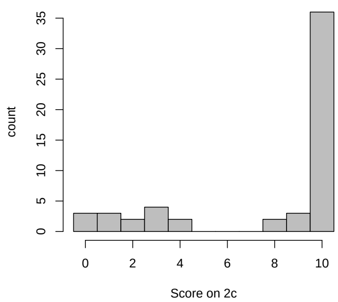
Score on 2b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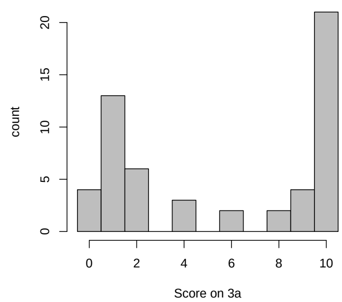
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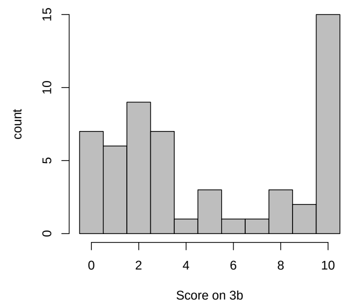
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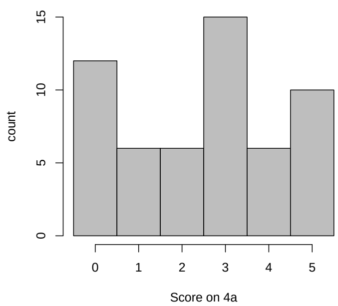
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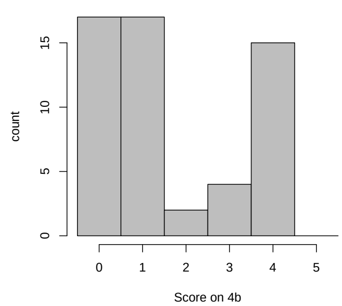
Score on 4a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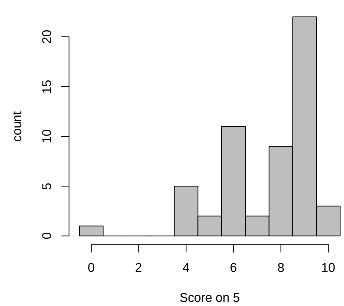
Score on 4b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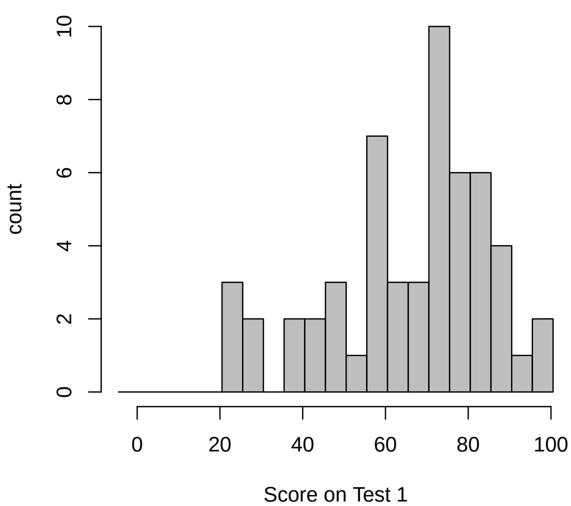
Score on 5


count
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Score on Test 1