score	possible	page
	30	1
	30	2
	20	3
	20	4
	100	

Name: ________________________________

Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student's solutions. (You may ask me questions.)

1. Determine whether each of the following statements is True or False.
Correct answers are worth +3, incorrect answers are worth −2, and no answer is worth +1.

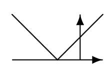
/3 (a) True False If $f(x) = (3x-1)^{-1}$ then $f(x+h) = \frac{1}{3x-1+3h}$.
True. $f(x+h) = \frac{1}{3(x+h)-1} = \frac{1}{3x-1+3h}$

/3 (b) True False If $x \neq 0$, $y \neq 0$, and $z \neq 0$, then $\dfrac{\frac{x}{y}}{z} = \dfrac{xz}{y}$.
False. $\dfrac{\frac{x}{y}}{z} = \dfrac{x}{yz}$.

/3 (c) True False $5 - \cos^2(\theta) - \sin^2(\theta) = 4$.
True. $\sin^2(\theta) + \cos^2(\theta) = 1$

/3 (d) True False If $x > 1$ then $\log_7(7^x) = x$.
True. Exponentials and logarithms are inverse functions. The assumption $x > 1$ is not needed.

/3 (e) True False If $x > 0$ then $\ln(x+2) - \ln(x) = \ln\left(1 + \dfrac{2}{x}\right)$.
True. $\ln(x+2) - \ln(x) = \ln\left(\dfrac{x+2}{x}\right) = \ln\left(1 + \dfrac{2}{x}\right)$.

/3 (f) True False The graph of $f(x) = |x+1|$ looks like 
True. The argument $x+1$ shifts the graph of $|x|$ to the left.

/3 (g) True False If $\lim_{x\to 3^+} f(x) = 4$ then $\lim_{x\to 3} f(x) = 4$.
False. If $\lim_{x\to 3^-} f(x) \neq 4$, then $\lim_{x\to 3} f(x)$ does not exist.

/3 (h) True False If $\lim_{x\to 3} f(x) = 4$ and $\lim_{x\to 3} g(x) = 5$, then $\lim_{x\to 3}(2f(x) + g(x)) = 13$.
True. Using the limit laws, $\lim_{x\to 3}(2f(x)+g(x)) = 2\left(\lim_{x\to 3} f(x)\right) + \left(\lim_{x\to 3} g(x)\right) = 2\cdot 4 + 5 = 13$.

/3 (i) True False If $x \neq 0$ then $\left(\dfrac{25}{4x^4}\right)\left(\dfrac{5}{2x^3}\right)^{-3} = \dfrac{2x^5}{5}$.
True. $\left(\dfrac{25}{4x^4}\right)\left(\dfrac{5}{2x^3}\right)^{-3} = \left(\dfrac{25}{4x^4}\right)\left(\dfrac{2x^3}{5}\right)^{3} = \left(\dfrac{25}{4x^4}\right)\left(\dfrac{2^3x^9}{5^3}\right) = \dfrac{25\cdot 2^3x^9}{4x^4\cdot 5^3} = \dfrac{2x^5}{5}$.

/3 (j) True False The equation $y-3 = 4(x-5)$ describes a line with slope 3 through the point $(4,5)$.
False, It is a line with slope 4 through the point $(5,3)$.

2. Compute the following limits. **Do not use L'Hôpital's rule!**

/10 (a) $\lim_{x\to 2} \dfrac{x-2}{x^2-5x+6} =$

$$\lim_{x\to 2} \frac{x-2}{(x-2)(x-3)} \qquad = \lim_{x\to 2} \frac{1}{x-3} \qquad = \frac{1}{2-3} = \frac{1}{-1} = -1\,.$$

/10 (b) $\lim_{h\to 0} \dfrac{h}{(2-h)^3-8} =$

$$\lim_{h\to 0} \frac{h}{(2^3+3(2^2(-h))+3(2(-h)^2)+(-h)^3)-8} \quad = \lim_{h\to 0} \frac{h}{-12h+6h^2-h^3} \quad = \lim_{h\to 0} \frac{h}{h(-12+6h-h^2)}$$
$$= \lim_{h\to 0} \frac{1}{-12+6h-h^2} \qquad = \frac{1}{-12+0+0} = \frac{1}{-12}\,.$$

/10 (c) $\lim_{x\to 9} \dfrac{x-9}{\sqrt{x}-3} =$

$$\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}\frac{\sqrt{x}+3}{\sqrt{x}+3} \qquad = \lim_{x\to 9} \frac{(x-9)(\sqrt{x}+3)}{x-9} \qquad = \lim_{x\to 9}(\sqrt{x}+3) \qquad = \sqrt{9}+3 = 6\,.$$

3. Compute the following limits. **Do not use L'Hôpital's rule!**

/10 (a) $\lim\limits_{x\to -3} \dfrac{x^{-1}+3^{-1}}{x+3} =$

$$\lim_{x\to -3} \frac{\frac{1}{x}+\frac{1}{3}}{x+3} = \lim_{x\to -3} \frac{\frac{3}{3x}+\frac{x}{3x}}{x+3} = \lim_{x\to -3} \frac{\frac{3+x}{3x}}{x+3} = \lim_{x\to -3} \frac{3+x}{3x(x+3)} = \lim_{x\to -3} \frac{1}{3x} = \frac{1}{3(-3)} = \frac{1}{-9}.$$

/10 (b) $\lim\limits_{\theta\to 0} \dfrac{\tan(7\theta)}{\sin(3\theta)} =$

$$\lim_{\theta\to 0} \frac{\frac{\sin(7\theta)}{\cos(7\theta)}}{\sin(3\theta)} = \lim_{\theta\to 0} \frac{\sin(7\theta)}{\cos(7\theta)\sin(3\theta)} = \lim_{\theta\to 0} \frac{1}{\cos(7\theta)} \frac{7}{3} \frac{\sin(7\theta)}{7\theta} \frac{3\theta}{\sin(3\theta)} = \frac{1}{1}\frac{7}{3}(1)(1) = \frac{7}{3}.$$

/5 4. (a) State the Squeeze Theorem using the template below.

If
- $f(x) \le g(x) \le h(x)$ when x is near a (except possibly at a) and
- $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$,

then $\lim_{x\to a} g(x) = L$.

/5 (b) Use the Squeeze Theorem to evaluate $\lim_{x\to 0} 2x^2 \sin\left(\frac{1}{x}\right)$.

Set

$$f(x) = -2x^2\,,$$
$$g(x) = 2x^2 \sin\left(\frac{1}{x}\right)\,, \quad \text{and}$$
$$h(x) = 2x^2\,.$$

Since $-1 \le \sin(\cdot) \le 1$ and $0 \le x^2$, we have $f(x) \le g(x) \le h(x)$ when x is near 0 (and for all $x \ne 0$). We can compute $\lim_{x\to 0} f(x) = \lim_{x\to 0} h(x) = 0$, so the assumptions of the Squeeze Theorem are satisfied and we can conclude $\lim_{x\to a} g(x) = 0$.

5. Sketch the graph of a single function f that has all of the following properties:

/2 (a) f has domain $[-3, 3]$.

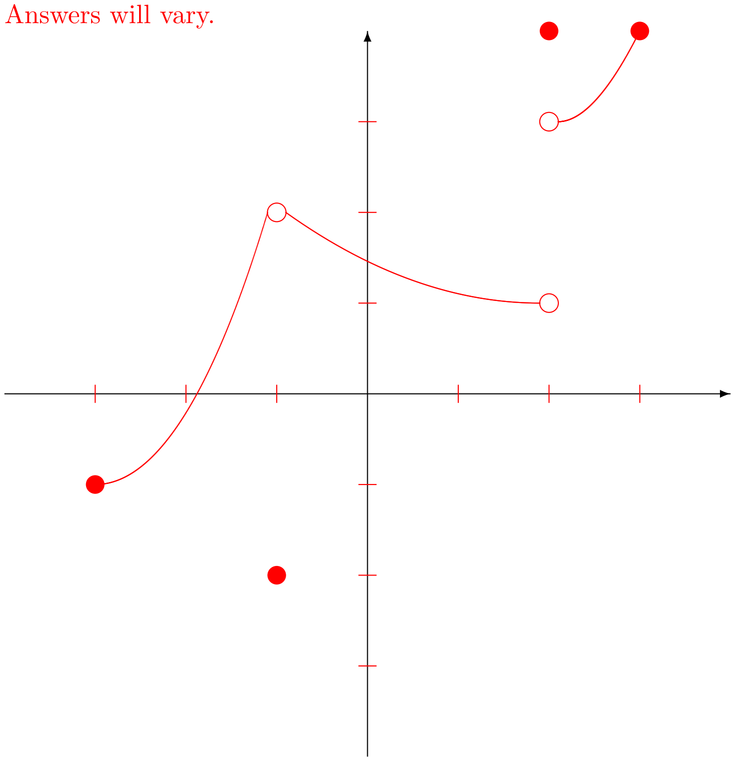
/2 (b) $\lim_{x\to 2^-} f(x) = 1$.

/2 (c) $\lim_{x\to 2^+} f(x) = 3$.

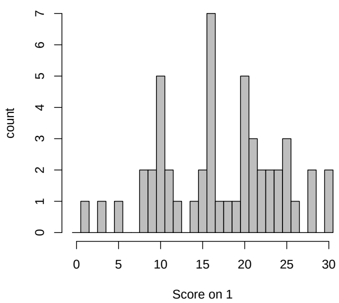
/2 (d) $\lim_{x\to -1} f(x) = 2$.

/2 (e) $f(-1) = -2$.

Answers will vary.



Scores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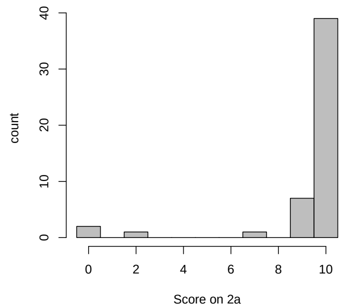
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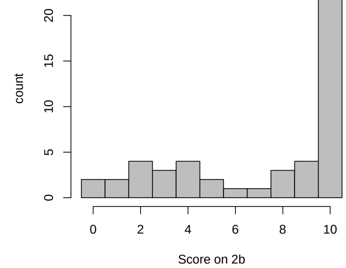
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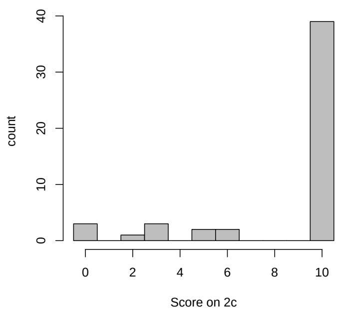
Score on 2b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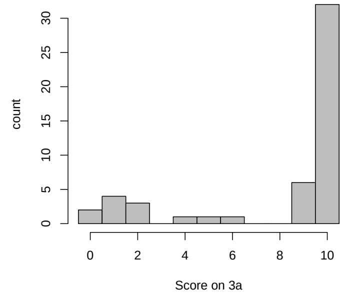
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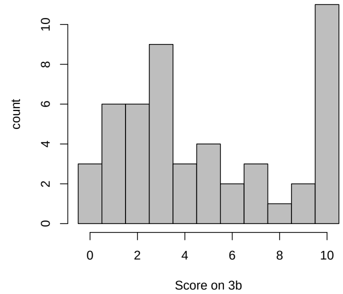
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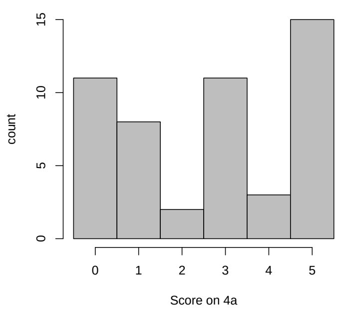
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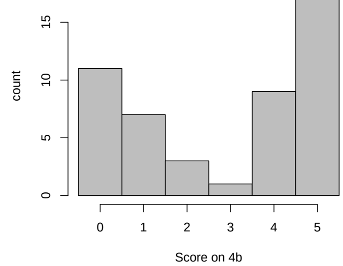
Score on 4a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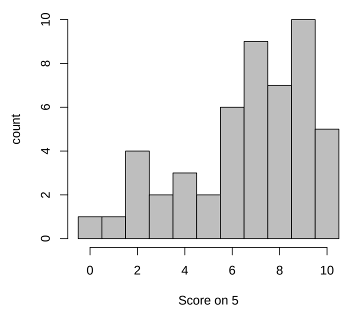
Score on 4b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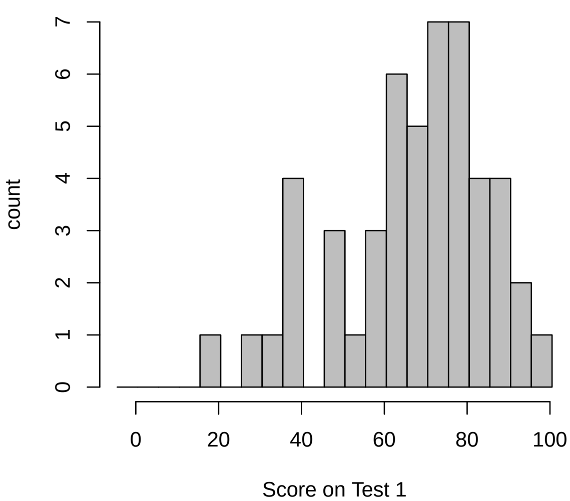
Score on 5


count
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Score on Test 1