score	possible	page
	20	1
	20	2
	30	3
	30	4
	100	

Name: ______________________________

Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student's solutions. (You may ask me questions.)

/10 1. Verify the identity $\sin(x) + \cot(x)\cos(x) = \csc(x)$.

Using $\cot(x) = \cos(x)/\sin(x)$ and $\csc(x) = 1/\sin(x)$ yields

$$\sin(x) + \frac{\cos(x)}{\sin(x)}\cos(x) = \frac{1}{\sin(x)}\,.$$

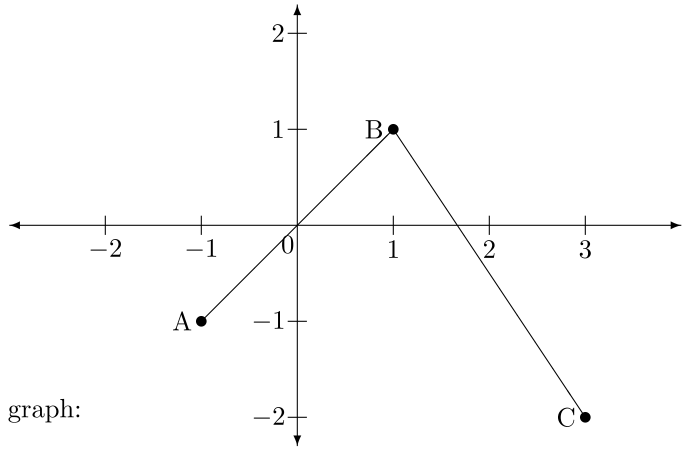
Multiplying by $\sin(x)$ yields $\sin^2(x) + \cos^2(x) = 1$, which we recognize as a basic trigonometric identity (or as a consequence of the Pythagorean theorem).

/10 2. Solve the following equation for x: $\log_2(x+2) - 2\log_2(x) = 0$.

$$\Rightarrow \log_2\left(\frac{x+2}{x^2}\right) = 0 \qquad \Leftrightarrow \frac{x+2}{x^2} = 1 \qquad \Leftrightarrow x+2 = x^2$$

$$\Leftrightarrow x^2 - x - 2 = 0 \qquad \Leftrightarrow (x-2)(x+1) = 0 \qquad \Leftrightarrow x = 2 \text{ or } -1\,.$$

Since the domain of $\log_2$ is $(0, \infty)$ and we started with $\log_2(x)$, we know $x > 0$ and so can eliminate $x = -1$ as a solution.



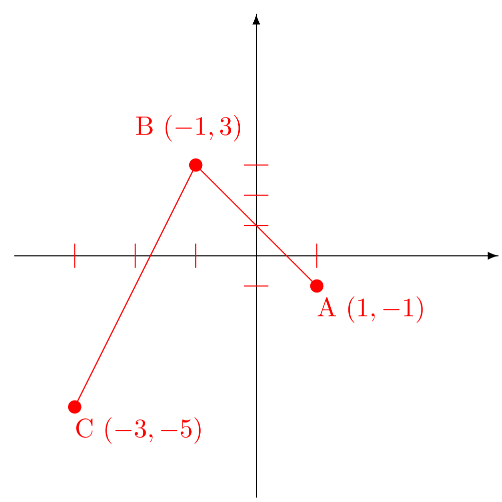
3. Consider $y = f(x)$ with graph:

/10 (a) Find the equations of the lines that comprise the graph of f.

On $[-1, 1]$ we use the point-slope form of a line to obtain $y - 1 = \frac{1--1}{1--1}(x-1) = x - 1$. Similarly, on $[1, 3]$ we obtain $y - 1 = \frac{1--2}{1-3}(x-1) = \frac{-3}{2}(x-1)$.

/10 (b) Draw the graph of $g(x) = 2f(-x) + 1$. Mark and label the points corresponding to A, B, and C.

For the reference point A, we want to use $f(-1)$, so we have to evaluate $g(1) = 2f(-1) + 1 = 2(-1) + 1 = -1$ and the point on the graph of g is $(1, -1)$. For B we have $g(-1) = 2f(1) + 1 = 2 \cdot 1 + 1 = 3$ and so $(-1, 3)$, and for C we have $g(-3) = 2f(3) + 1 = (-2)3 + 1 = -5$ and so $(-3, -5)$.



4. Consider the rational function $f(x) = \dfrac{x^2 - x - 6}{(x+1)(x+3)}$.

/6 (a) Express the domain of f in interval notation.

$f(x) = \frac{(x+2)(x-3)}{(x+1)(x+3)}$. Since we divide by 0 at $x = -3$ and $x = -1$, the domain is $(-\infty, -3) \cup (-3, -1) \cup (-1, \infty)$.

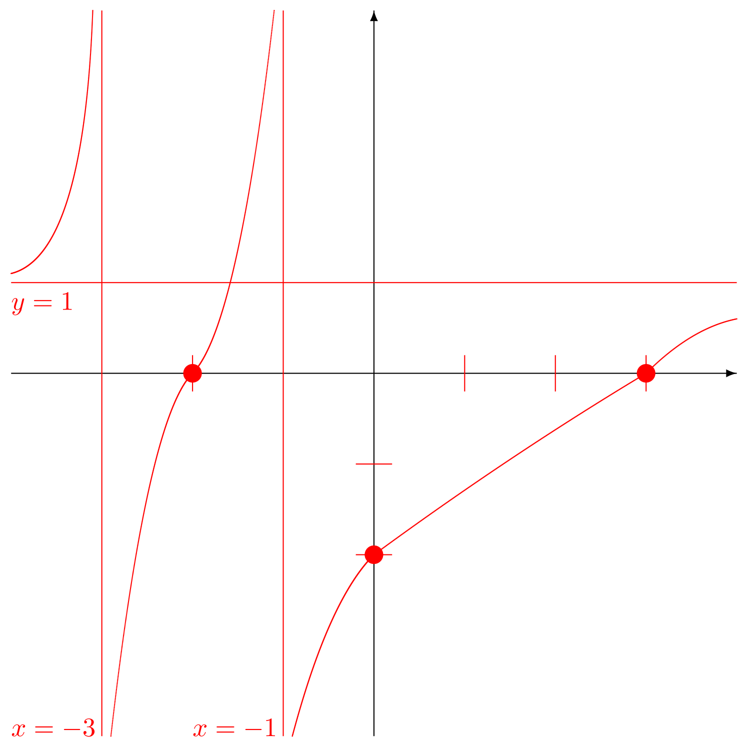
/6 (b) Find the x and y intercepts of f.

$f(0) = \frac{(0+2)(0-3)}{(0+1)(0+3)} = -2$ so the y-intercept is at $(0, -2)$. Setting $0 = (x+2)(x-3)$ yields $x = -2$ and $x = 3$, so the x-intercepts are at $(-2, 0)$ and $(3, 0)$.

/8 (c) Find all vertical and horizontal asymptotes and identify any holes.

Since there are no common factors in the numerator and denominator, there are no holes. There are vertical asymptotes where we divide by zero, at $x = -3$ and $x = -1$. Horizontal asymptotes are determined by the highest powers in the numerator and denominator, so we have $\frac{x^2-x-6}{x^2+4x+3} \to \frac{x^2}{x^2} = 1$ and $y = 1$ is a horizontal asymptote.

/10 (d) Sketch a detailed graph of f.



5. Simplify and cancel so that you can plug in the given value without dividing by 0.

/10 (a) For $x = 9$, $\dfrac{x-9}{\sqrt{x}-3} =$

$$\frac{x-9}{\sqrt{x}-3}\frac{\sqrt{x}+3}{\sqrt{x}+3} \quad = \frac{(x-9)(\sqrt{x}+3)}{x-9} \quad = \sqrt{x}+3 \quad = 6\,.$$

(Fishy since we used both $x \neq 9$ and $x = 9$.)

/10 (b) For $h = 0$ and $f(x) = 2x^2$, $\dfrac{f(x+h)-f(x)}{h} =$

$$\frac{2(x+h)^2 - 2x^2}{h} \quad = \frac{2(x^2+2xh+h^2-x^2)}{h} \quad = \frac{2(2xh+h^2)}{h}$$

$$= \frac{2h(2x+h)}{h} \quad = 4x+2h \quad = 4x\,.$$

(Fishy since we used both $h \neq 0$ and $h = 0$.)

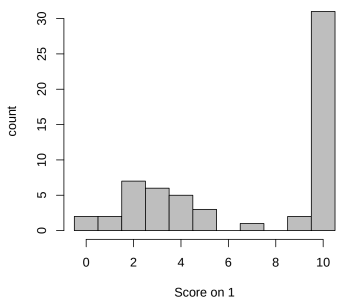
/10 (c) For $h = 0$, $\dfrac{(x+h)^{-1} - x^{-1}}{h} =$

$$\frac{\frac{1}{(x+h)} - \frac{1}{x}}{h} \quad = \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h} \quad = \frac{\frac{x-(x+h)}{x(x+h)}}{h}$$

$$= \frac{\frac{-h}{x(x+h)}}{h} \quad = \frac{-h}{hx(x+h)} \quad = \frac{-1}{x(x+h)} \quad = \frac{-1}{x^2}\,.$$

(Fishy since we used both $h \neq 0$ and $h = 0$.)

Scores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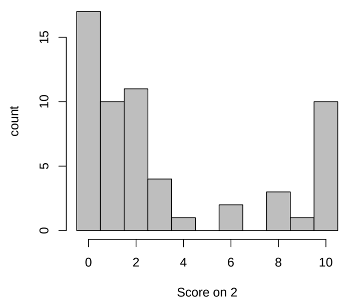
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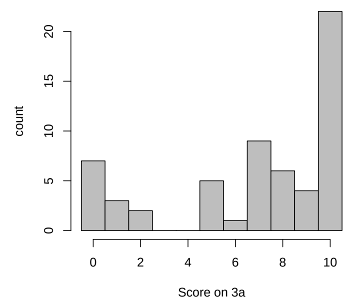
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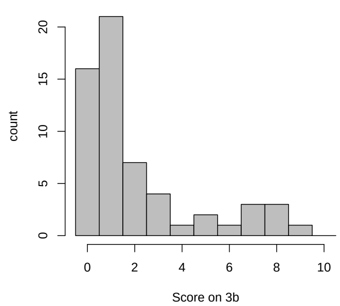
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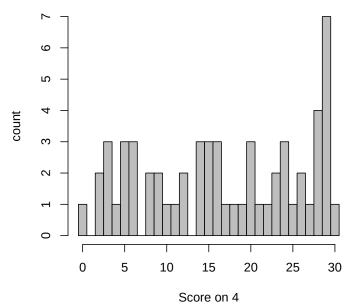
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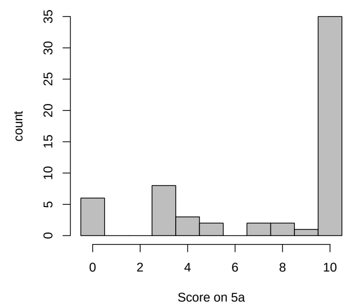
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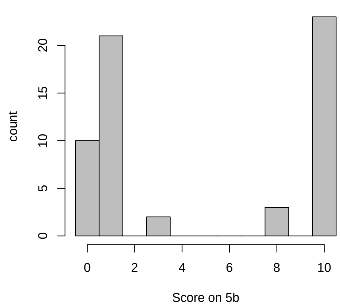
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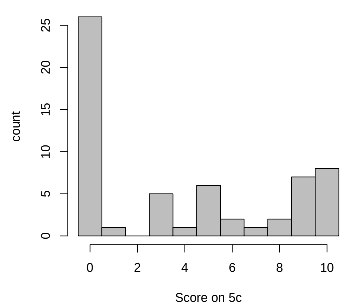
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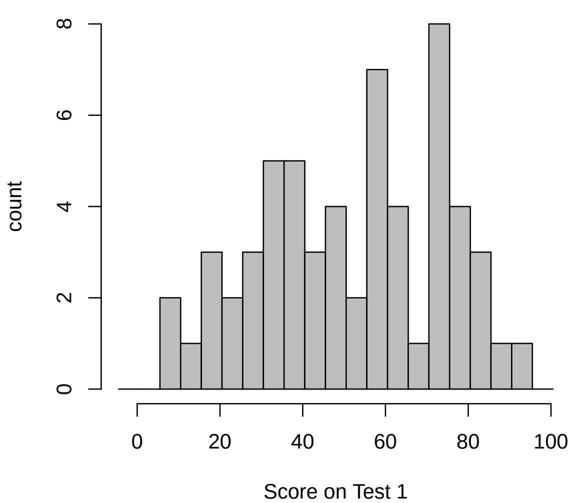
Score on 5c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Score on Test 1