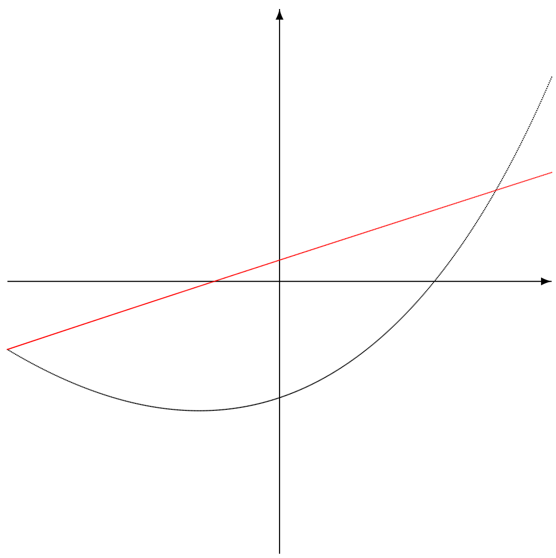
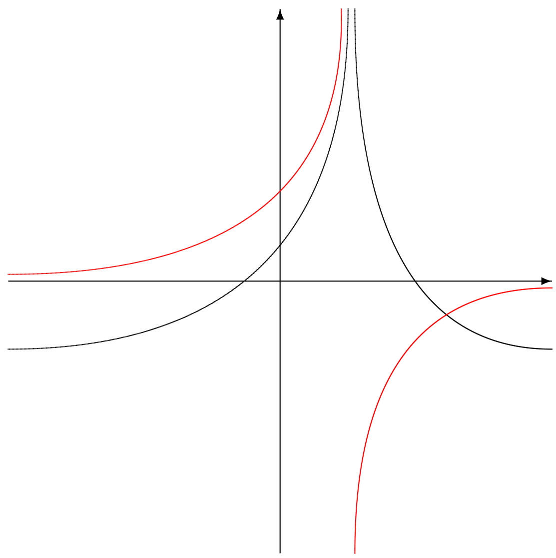
score	possible	page
	20	1
	30	2
	20	3
	30	4
	100	

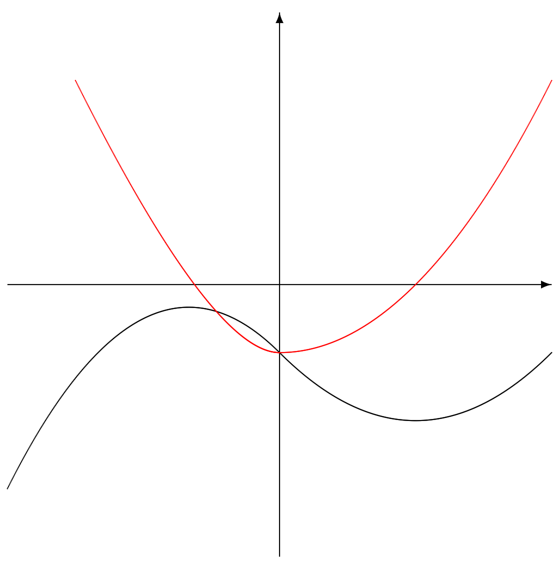
Name: ________________________________

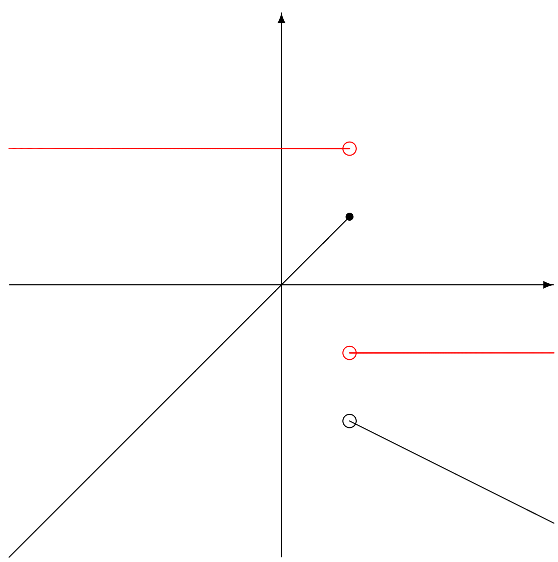
Show your work!
You may not give or receive any assistance during a test, including but not limited to using notes, phones, calculators, computers, or another student's solutions. (You may ask me questions.)

/20 1. The graph of a function f is given in each part below. On the same axes, sketch the graph of f'.









2. Let $f(x) = (2x+1)^{-1}$.

/10 (a) Using the definition of the derivative as a limit, compute $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(2(x+h)+1)^{-1} - (2x+1)^{-1}}{h} = \lim_{h\to 0} \frac{\frac{1}{(2(x+h)+1)} - \frac{1}{(2x+1)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(2x+1)-(2(x+h)+1)}{(2x+1)(2(x+h)+1)}}{h} = \lim_{h\to 0} \frac{-2h}{h(2x+1)(2(x+h)+1)}$$

$$= \lim_{h\to 0} \frac{-2}{(2x+1)(2(x+h)+1)} = \frac{-2}{(2x+1)(2(x+0)+1)} = \frac{-2}{(2x+1)^2}\,.$$

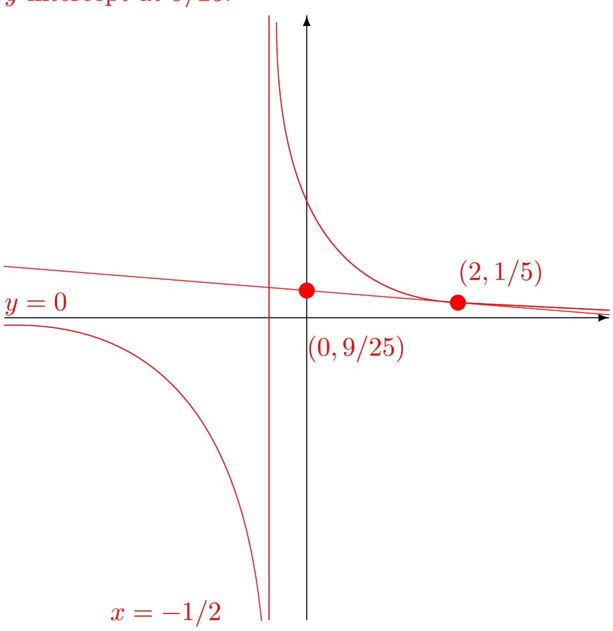
/10 (b) Find the equation for the tangent line at $x = 2$.

The general form of the tangent line at A is $y = f'(A)(x-A) + f(A)$ so we have

$$y = f'(2)(x-2) + f(2) = (-2/25)(x-2) + 1/5\,.$$

/10 (c) Graph $f(x)$ and the tangent line.

There is a vertical asymptote at $x = -1/2$ and a horizontal asymptote at $y = 0$. Near the vertical asymptote, $\lim_{x\to(-1/2)^+} f(x) = \infty$ and $\lim_{x\to(-1/2)^-} f(x) = -\infty$. The tangent line has y-intercept at $9/25$.



/10 3. Compute the following limits and simplify the results.

(a) $\lim_{x\to\infty} \pi/x =$

$$0$$

(b) $\lim_{x\to\infty} \cos(\pi/x) =$

$$\cos(0) = 1$$

(c) $\lim_{x\to\infty} \sin\left(\frac{\pi}{6\cos(\pi/x)}\right) =$

$$\sin\left(\frac{\pi}{6(1)}\right) = \frac{1}{2}$$

(d) $\lim_{x\to\infty} \log_2\left(\sin\left(\frac{\pi}{6\cos(\pi/x)}\right)\right) =$

$$\log_2\left(\frac{1}{2}\right) = -1$$

/10 4. State

- the definition of "Continuous" and
- the definition of "Differentiable".

Give an example of a function that is one but not the other.

- A function f is continuous at a if
$$\lim_{x\to a} f(x) = f(a)\,.$$
- A function f is differentiable at a if $f'(a)$ exists.

The function $f(x) = |x|$ is continous (everywhere), but is not differentiable at $a = 0$ since

$$\lim_{h\to 0^-} \frac{|0+h|}{h} = \lim_{h\to 0^-} \frac{-h}{h} = -1 \neq \lim_{h\to 0^+} \frac{|0+h|}{h} = \lim_{h\to 0^-} \frac{h}{h} = 1\,.$$

5. Compute the following derivatives:

/10 (a) $f(x) = 2 + x - x^2 + \dfrac{3}{x} - \sqrt{x} - 5x^7 + x^{3/4} + 3\sin(x) + \cot(x) - \sin(7)$
$\Rightarrow f'(x) =$

$$f'(x) = 0 + 1 - 2x - 3x^{-2} - \frac{1}{2\sqrt{x}} - 35x^6 + \frac{3}{4}x^{-1/4} + 3\cos(x) - \csc^2(x) - 0$$

/10 (b) $D_x\left[\cos(x)(8\cos(x) + x^5 + 3x)\right] =$

$$-\sin(x)(8\cos(x) + x^5 + 3x) + \cos(x)(-8\sin(x) + 5x^4 + 3)$$

/10 (c) $y = \dfrac{x^3 - 5x}{\sin(x) + x} \Rightarrow \dfrac{dy}{dx} =$

$$\frac{(3x^2 - 5)(\sin(x) + x) - (x^3 - 5x)(\cos(x) + 1)}{(\sin(x) + x)^2}$$