The tests are cummulative and can include Pre-Calculus material mentioned in the MATH 2301 Calculus I handbook. This guide gives some sample questions for Sections 4.1, 4.2, 4.3, and 4.4. In some cases part of the problem is deciding which method to use, so you may be able to do the problem using methods from earlier sections. Doing these problems does not replace doing homework problems.

1. Find the absolute maximum and minimum values of f on the given interval.

$$f(t) = t\sqrt{4-t^2} \quad \text{on} \quad [-1, 2]$$

2. State the Rolle's Theorem. Identify what are its assumptions (hypotheses) and what are its conclusions.

3. Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

$$f(x) = 5 - 12x + 3x^2 \quad \text{on} \quad [1, 3]\,.$$

4. Let f be a continuous function with $f(0) = 3$, $f(2) = 6$, $f'(x) = 0$ for $0 < x < 1$, and $f'(x) < 2$ for $1 < x < 2$. Sketch such a function or explain why it is impossible.

5. State the Mean Value Theorem (MVT). State why the function

$$f(x) = x^3 - 3x + 2 \quad \text{on the interval} \quad [-2, 2]$$

satisfies each of the hypotheses of the MVT on the given interval. Then find all numbers c that satisfy the conclusion of the MVT.

6. Let $f(x) = \ln(1 + x^2)$.
   (a) Find the intervals where f is increasing, and the intervals where it is decreasing.
   (b) Find the intervals where f is concave up, and the intervals where it is concave down.

7. Let $f(x) = x^3 - 3x^2 - 45x + 2$.
   (a) Find the intervals where f is increasing, and the intervals where it is decreasing.
   (b) Find the intervals where f is concave up, and the intervals where it is concave down.
   (c) Find the absolute maximum and minimum values of f on the interval $[0, 1]$.

8. For the function

$$f(x) = xe^{-x}$$

   (a) Find any vertical asymptotes.
   (b) Find the intervals on which f is increasing or decreasing.
   (c) Find the local maximum and minimum values of f.
   (d) Find the intervals of concavity and the inflection points.
   (e) Use the information from (a)-(d) to sketch the graph.

9. Analyze and graph the function $f(x) = x + \dfrac{9}{x}$.

10. For the function $f(x) = \dfrac{x-1}{x^2}$,
    (a) Find the horizontal and vertical asymptotes.
    (b) Find the intervals on which f is increasing or decreasing.
    (c) Find the local maximum and minimum values of f.
    (d) Find the intervals of concavity and the inflection points.
    (e) Use the information from (a)-(d) to sketch the graph.

11. Sketch the graph of a single function that has all of the following properties:
    (a) Continuous and differentiable everywhere except at $x = -3$, where it has a vertical asymptote.
    (b) A horizontal asymptote at $y = 1$.
    (c) An x-intercept at $x = -2$.
    (d) A y-intercept at $y = 4$.
    (e) $f'(x) > 0$ on the intervals $(-\infty, -3)$ and $(-3, 2)$.
    (f) $f'(x) < 0$ on the interval $(2, \infty)$.
    (g) $f''(x) > 0$ on the intervals $(-\infty, -3)$ and $(4, \infty)$.
    (h) $f''(x) < 0$ on the interval $(-3, 4)$.
    (i) $f'(2) = 0$.
    (j) An inflection point at $(4, 3)$.