Work in groups of 3 or 4. I recommend splitting up the problems and then checking each other's work. Show your work. Aknowledge any help on these specific problems.

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Final - MATH 2301 Calculus I - Spring 2014

All problems are worth the same number of points. Partial credit will be given for correct work. Do not simplify your answers unless requested.

/5 1. Sketch the graph of a function f that:
has a local maximum at $x = -1$ but is not differentiable there,
$\lim_{x\to 1} f(x) = +\infty$, but is continuous elsewhere,
$\lim_{x\to+\infty} f(x) = -1$ and $f''(x) > 0$ everywhere it exists.

/5 2.

(a) Find the limit : $\lim\limits_{h\to 0} \dfrac{(2+h)^5 - 2^5}{h}$. (Don't use l'Hopital's rule.)

(b) What derivative does the limit represent?

/5 3. Find dy/dx by implicit differentiation and simplify if possible.
$\tan(x/y) = x^2 + y^2$

/5 4. Find the derivative of the function: $y = \sqrt{3x}$.

(a) Using the definition of derivative (i.e. a limit).

(b) Using differentiation rules.

/5 5. Find the derivative of the function:
$f(x) = \cosh(3x - 1) + x \tan x.$

/5 6. (a) Find the intervals on which f is increasing or decreasing.
(b) Find the local maximum and minimum values of f.
(c) Find the intervals of concavity and the inflection points.
(d) Find any horizontal or vertical asymptotes.
(e) Use the information from (a)-(d) to sketch the graph.

$f(x) = \dfrac{x}{x^2 + 9}.$ Hint: $f''(x) = \dfrac{2x(x^2 - 27)}{(x^2 + 9)^3}$

/5 7. Find the derivatives of the functions:

(a) $f(x) = \tan^{-1}(\cos x)$, (b) $g(x) = \ln x^2$

/5 8. A plane flying horizontally, in a straight line, at an altitude of 1 km and a speed of 500 km/hr passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 km away from the station.

/5 9. Find the derivatives of the functions:

(a) $f(t) = \sin^{-1}(t)$ (b) $g(t) = \int_0^t e^{-x^2}\,dx$

/5 10. If $f(1) = 231$ and $f'(x) \leq -5.8$, what is the largest possible value of $f(6)$? Draw a diagram illustrating why your answer is correct.

/5 11. Find an equation of the tangent line to the graph of $y = 4e^{x/2}$ at $x = 2$.

/5 12. Find $\int \sin^2 x \cos x \, dx$.

/5 13. A Norman window has the shape of a rectangle surmounted by a semicircle. (The diameter of the semicircle is the width of the rectangle.) If the perimeter of the window is to be 30 ft, find the dimensions of the window with the largest possible area.

/5 14. Find the absolute maximum and minimum values of the function:
$f(x) = 3x^4 - 4x^3 - 12x^2 + 1$ on the interval $[-2, 3]$.

/5 15.

(a) Find the average value of the function $f(x) = (x-3)^2$ on the interval $[2, 5]$.

(b) Find c such that $f(c) = f_{ave}$

/5 16. Use Newton's method with the specified initial approximation x_1 to find x_2, the second approximation to the root of the given equation. Leave the answer as a fraction.

$x^7 + 4 = 0\ , \quad x_1 = -1.$

/5 17. Evaluate the integral by interpreting it in terms of areas: $\int_{-2}^{2} \left(1 + \sqrt{4 - x^2}\right)\, dx.$

/5 18. 4-methylcyclohexanemethanol (MCHM) leaked from a tank at a rate of $r(t)$ liters (l) per hour (h). The rate decreased as time passed and the values of the rate at two-hour intervals are shown in the table. Find lower and upper estimates for the total amount of MCHM that leaked out. What is the integral that expresses this amount?

t (h)	0	2	4	6	8	10
$r(t)$ (l/h)	8.7	7.6	6.8	6.2	5.7	5.3

/5 19. Find the value of the integral:

$$\int_0^{\frac{\sqrt{3}}{2}} \frac{dr}{\sqrt{1-r^2}}$$

/5 20. Find an antiderivative of the function: $g(x) = 2\sqrt[3]{x} - e^{2x}$ and check by differentiating.