score	possible	problem
	20	1
	20	2
	30	3
	30	4
	100	

Name: ______________________________
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Work in groups of 3 or 4. Show your work. Aknowledge any help on these specific problems.

/20 1. Use the formula

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{\infty} f(x_i)\Delta x$$

with $\Delta x = (b-a)/n$ and $x_i = a + i\Delta x$ to evaluate the integral

$$\int_0^1 (x^3 - 3x^2)dx\,.$$

/20 2. Suppose f and g are differentiable functions with the following properties:

$$
\begin{array}{lll}
f(0) = 2 & f(1) = 0 & f(2) = 1 \\
g(0) = 1 & g(1) = 2 & g(2) = 0 \\
\int_0^1 f(x)dx = \pi & \int_1^2 f(x)dx = \pi^3 & \int_2^3 f(x)dx = \pi^5 \\
\int_0^1 g(x)dx = \sqrt{2} & \int_1^2 g(x)dx = \sqrt{3} & \int_2^3 g(x)dx = \sqrt{5} \\
f'(0) = e & f'(1) = e^3 & f'(2) = e^5 \\
g'(0) = \sqrt{7} & g'(1) = \sqrt{11} & g'(2) = \sqrt{13}
\end{array}
$$

Evaluate the following. If one cannot be evaluated with the given information, write "NOT ENOUGH INFORMATION."

(a) $\int_0^1 f(r)dr$

(b) $\int_0^3 f(x)dx$

(c) $\int_3^2 g(x)dx$

(d) $\int_1^2 (5f(x) + g(x))dx$

(e) $\int_0^1 f(x)g(x)dx$

(f) $\int_0^{14} f(x)dx - \int_2^{14} f(x)dx$

(g) $\int_0^2 f'(r)\, dr$

(h) $\int_6^6 f''(x)\, dx$

(i) $\displaystyle\lim_{x \to 1} \frac{f(x)}{g(x) - 2}$

(j) $\displaystyle\lim_{h \to 0} \frac{f(2+h) - 1}{h}$

/10 3. (a) Evaluate the integral $\int_{-1}^{2} |x| dx$ by interpreting it in terms of area.

/10 (b) If $f(0) = 0$ and $3 \le f'(x) \le 5$, what is the smallest that $\int_{0}^{4} f(x)\, dx$ can be? What is the largest it can be?

/10 (c) Evaluate the integral $\int_{-1}^{2} x^3 dx$ and interpret it as a difference of areas. Illustrate with a sketch.

4. Evaluate the integrals:

/5 (a) $\displaystyle\int_{-2}^{3}(x^2-3)dx =$

/5 (b) $\displaystyle\int_{1}^{4}\left(\frac{4+6u}{\sqrt{u}}\right)du =$

/5 (c) $\displaystyle\int_{1}^{2}\left(\frac{x}{2}-\frac{2}{x}\right)dx =$

/5 (d) $\displaystyle\int_{1}^{e}\frac{x^2+x+1}{x}dx =$

/5 (e) $\displaystyle\int\frac{\sin(x)}{1-\sin^2(x)}dx =$

/5 (f) $\displaystyle\int\tan(7)dx =$