

score	possible	problem
	25	1
	25	2
	25	3
	25	4
	100	

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Work in groups of 3 or 4. Show your work. Acknowledge any help on these specific problems.

/25

1. A company wishes to manufacture a box with a volume of 6 m^3 that is open on top and has a square base. The material for the bottom of the box costs \$3 per m^2 , while the material for the sides costs \$2 per m^2 . Find the dimensions of the box that will lead to minimum total cost. What is the minimum total cost?

2. For integers $m \leq n$ and some numbers $\{a_i\}$, sigma notation is defined by

$$\sum_{i=m}^n a_i = a_m + a_{m+1} + \cdots + a_n.$$

The following are proposed properties of sigma notation. For each of them, either

- explain/ prove/ show that it is (always) true or
- give an example showing that it is false.

/3 (a) $\sum_{i=m}^n a_i = \sum_{j=m}^n a_j$

/3 (b) $\sum_{i=m}^n a_i = \sum_{i=0}^{n-m} a_{i-m}$

/3 (c) $\sum_{i=m}^n (3a_i) = 3 \left(\sum_{i=m}^n a_i \right)$

/3 (d) $\sum_{i=m}^n a_{3i} = \sum_{i=3m}^{3n} a_i$

/3 (e) $\sum_{i=m}^n (a_i + b_i) = \left(\sum_{i=m}^n a_i \right) + \left(\sum_{i=m}^n b_i \right)$

/3 (f) $\sum_{i=m}^n (a_i b_i) = \left(\sum_{i=m}^n a_i \right) \left(\sum_{i=m}^n b_i \right)$

/3 (g) $\sum_{i=m}^n |a_i| = \left| \sum_{i=m}^n a_i \right|$

/4 (h) $\sum_{i=1}^n i = \frac{n(n+1)}{2}$

- /10 3. 4-methylcyclohexanemethanol (MCHM) leaked from a tank at a rate of $r(t)$ liters (l) per hour (h). The rate decreased as time passed and the values of the rate at two-hour intervals are shown in the table. Find lower and upper estimates for the total amount of MCHM that leaked out.

$t(h)$	0	2	4	6	8	10
$r(t)$ (l/h)	8.7	7.6	6.8	6.2	5.7	5.3

- /15 4. A brick falls from the top of a tall building 450m above the ground.
- (a) Find the distance of the brick above ground level at time t .
 - (b) How long does it take the brick to hit the ground?
 - (c) With what velocity does it hit the ground?
 - (d) If it had been thrown down with an initial velocity of 5m/s, how long would it take to reach the ground?

/25 5. For the function

$$f(x) = \frac{x^2}{x^2 + 9}$$

- (a) Find the x - and y -intercepts.
- (b) Find any asymptotes.
- (c) Find the intervals on which f is increasing or decreasing.
- (d) Find the local maximum and minimum values of f .
- (e) Find the intervals of concavity and the inflection points.
- (f) Use the information above to sketch the graph.

