Here are some sample questions, mostly from old tests. Most of the final exam questions on Part IV will be quite similar to these, but other topics that we covered are still fair game.

1. Write the IVP: $\theta'' + .5\theta' + \sin\theta = \sin 2t$, $\theta(0) = 1$, $\theta'(0) = 0$ as a system of first order equations. Give all the MATLAB commands needed to solve this IVP on the interval $0 \le t \le 10$.

2. Write a well-commented MATLAB **function** program to do n steps of the Euler method for a differential equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$, on the time interval $[a, b]$ with $\mathbf{x}(a) = \mathbf{x}_0$. Let the first line be:
   `function [T, X] = myeuler(f,x0,a,b,n)`.

3. Describe RK45. What is the command for it in MATLAB?

4. What is variable step size? How is it implemented RK45?

5. (a) Write a well-commented MATLAB **function** program to do n steps of the modified Euler method for a differential equation $\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x})$, on the time interval $[a, b]$ with $\mathbf{x}(a) = \mathbf{x}_0$. Let the first line be:
   `function [T, X] = mymodeuler(f,x0,a,b,n)`.

   (b) Give the sequence of MATLAB commands needed to use your function `mymodeuler` to solve the ODE $s''(r) = \cos(r) + s'(r)$ with $s(3) = 5$ and $s'(3) = 4$ on the interval $3 \le r \le 11$.

   (c) In what way(s) is MATLAB's `ode45` function better than your function `mymodeuler`?

6. (a) Derive the explicit finite difference equations for solving the heat/diffusion equation $u_t = cu_{xx}$ on the interval $x \in [0, L]$ with boundary conditions $u(0, t) = a$, $u(L, t) = b$, and $u(x, 0) = f(x)$.

   (b) Explain how to incorporate an insulated boundary at $x = L$.

7. (a) Derive the explicit finite difference equations for solving the heat/diffusion equation $u_t = cu_{xx}$ on the interval $x \in [0, L]$ with boundary conditions $u(0, t) = a$, $u(L, t) = b$, and $u(x, 0) = f(x)$.

   (b) When and why does the explicit finite difference method for the heat/diffusion equation become unstable?

   (c) Derive the implicit finite difference equations for solving the heat/diffusion equation $u_t = cu_{xx}$.

8. For the heat/diffusion equation $u_t = cu_{xx}$ we learned an explicit method, an implicit method, and (in less detail) the Crank-Nicholson method.

   - Compare and contrast the explicit and implicit methods. Which would you recommend, and why?
   - What is the Crank-Nicholson method? Would you recommend it over the other two? Why?

9. (a) Set up the finite difference equations for the BVP: $u_{xx} + u_{yy} = f(x, y)$, on the rectangle $0 \leq x \leq a$ and $0 \leq y \leq b$, with $u = 0$ on all the boundaries. Explain how the difference equations could be solved as a linear system.

   (b) Explain how to incorporate an insulated boundary at $x = a$.

10. A finite element solution to a differential equation in two dimensions is of the form

$$U(x, y) = \sum_{j=1}^{n} C_j \Phi_j(x, y)\,.$$

   (a) Explain what the Φ_j are and how you construct/ find/ solve for them.

   (b) Explain what the C_j are and how you construct/ find/ solve for them.

11. Explain why order matters in engineering problems.