

Here are some sample questions, mostly from old tests. Most of the test will be quite similar to these, but other topics that we covered are still fair game.

1. What is the condition number of a matrix? How do you find it with MATLAB? What are the implications of the condition number when solving a linear system? What is the engineering solution to a problem with a bad condition number?
2. (a) Using pivoting if appropriate, find the LU decomposition of

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}.$$

(b) Using your LU decomposition, solve $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$.

3. Write a well-commented MATLAB **function** program that solves the linear systems $A\mathbf{x}_1 = \mathbf{b}_1$ and $A\mathbf{x}_2 = \mathbf{b}_2$ using LU decomposition. Let A , \mathbf{b}_1 , and \mathbf{b}_2 be the inputs and \mathbf{x}_1 and \mathbf{x}_2 be the outputs.
4. Write a well-commented MATLAB **function** program to that solves a linear system $A\mathbf{x} = \mathbf{b}$ using LU decomposition. Let A , \mathbf{b} and tol be the inputs and \mathbf{x} the output. If the error (residual) is not less than tol , then display a warning.
5. Write a well-commented MATLAB **function** program that takes an input n , produces a random $n \times n$ matrix A and random vector $\bar{\mathbf{b}}$, solves $A\bar{\mathbf{x}} = \bar{\mathbf{b}}$ (using the built in command) and outputs the residual (number).
6. Write a well-commented MATLAB **script** program that will use Newton's method to find a solution to the system of equations

$$\begin{aligned} x^3 - 7y^2 &= 1 \\ 5y^3 &= \cos(x) + 2 \end{aligned}$$

starting from the initial guess $(x, y) = (11, 17)$.

7. Write a well-commented MATLAB **script** program that will use Newton's method to find a solution to the system of equations

$$\begin{aligned} x^3 - 7y^2 + 1 &= 0 \\ 5y^3 + x + 2z - 2 &= 0 \\ 5z^3 + x - y^2 - 3 &= 0 \end{aligned}$$

starting from the initial guess $(x, y, z) = (1, 1, 1)$.

8. (a) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -2 & -9 \\ -1 & -2 \end{bmatrix}.$$

- (b) Perform one iteration of the power method, starting with $\begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$. What approximate eigenvalue and eigenvector did you get? If you kept iterating, what would you get eventually?
- (c) Perform one iteration of the inverse power method, starting with $\begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$. What approximate eigenvalue and eigenvector did you get? If you kept iterating, what would you get eventually?
9. Write a well-commented MATLAB **function** program to do n iterations of the Power Method. Let the matrix A and n be inputs and let $[\mathbf{e} \ \mathbf{v}]$ (the eigenvalue and eigenvector) be the outputs.