IV Teaching and Advising

IV.A Philosophy of Teaching

Learning to teach

After teaching a few courses, I realized that learning to teach well is an ongoing process. Although I believe that I am an effective teacher, I realize that I will always need improvement, so I continue to develop my abilities as a teacher. At the formal level, my training has consisted of various education seminars on campus and conferences off campus:

- U. of Colorado Teaching with Technology conference, U. of Colorado at Boulder, Aug. 2004
- Seminar/Workshop: Teaching Writing in the Discipline (WID), Ohio U., Jan. 2004
- CITL114: Cooperative/Collaborative Learning workshop, Ohio U., Oct. 2002
- U. of Colorado Teaching with Technology conference, U. of Colorado at Boulder, Jun. 2002
- U. of Colorado Teaching with Technology conference, U. of Colorado at Colorado Springs, Jul. 2000
- U. of Colorado Teaching with Technology conference, Colorado School of Mines, Jul. 1999
- Graduate Teacher Program/ Colorado Preparing Future Faculty Network conference, U. of Colorado, Jan. 1999

At a less formal level, I am an active participant in the Math Education seminar. Although I am not interested in doing research in Math education myself, I am quite interested in what the research has to say about the effectiveness of various techniques. At the informal level, I am always on the lookout for effective techniques used by my colleagues.

A few times I thought I had discovered the key to effective student learning. With time, however, that technique faded to become simply part of my repertoire. Now I understand that learning is a very messy process, and there is no secret key. When I encounter an interesting insight or promising technique, I experiment with it to see if it can enhance my overall effectiveness. Through such experimentation I have acquired many techniques, but I have also acquired an appreciation of cost-benefit analysis. Various 'reform' techniques appear valuable, but require a great deal of time and effort to use. I find that many of their benefits can be achieved with a far simpler implementation of their ideas.

Aspects of my teaching method that may be unusual, and their rational

Student learning takes place in the student's head, to which the instructor only has limited access. Although I think I can do a fine job at lecturing, I still see the students' eyes glazing over, and then I start wondering what the point of lecturing is. All the information they need is written down very clearly in the textbook, with many illustrations and examples. If I develop a more exciting lecture style I may prevent glazed eyes, and help my ego, but that does not mean the students are learning more. Because of these considerations, I have been trying to reduce the amount of time that I am talking and increase the amount of time the students are actively learning. As an important side effect, this approach shifts the students' focus away from the instructor and toward the material, and thus moves them toward being independent learners. In an introductory class, such as Calculus, I now typically lecture 10-15 minutes to give them the main idea and one example, and then give them a couple of similar problems to work on. They are encouraged to work in groups, and I circulate to answer questions. I then write the solution on the board, take any further questions, and the cycle repeats. In the class MATH 344 *Numerical Methods for Civil and Mechanical Engineers* I hardly lecture at all. The class is taught in a computer lab, using the software package Matlab. The textbook is online and contains all the exercises. I typically talk for less than ten minutes to give the big picture, and then set them loose on the material, while I circulate to answer questions. In graduate classes I often divide the material for that day among the students, give them a few minutes to prepare, and then have them present the material. Although their presentations are not as polished as mine would be, this method often exposes difficulties and misunderstandings that I would have missed.

Teaching a topic is a good way to learn it. The very act of explaining seems to stimulate one's brain to understand better. As mentioned above, students in my graduate classes present much of the material themselves. Students in my undergraduate classes are are given the opportunity to explain things to each other when they work in groups on problems. I also use writing assignments, as described in detail in Section IV.B. In these exercises, the students explain in writing how they solved the problem, and the reasoning they used. Many students have told me that these assignments really helped them to understand important mathematical concepts.

There are several different styles (visual, aural, read/write, kinesthetic) in which students may learn. I try to mix these aspects into the classroom/course. However, I think it is more impactful for each student to be aware of their own learning style, so they can adapt and compensate as needed. I try to accomplish this by having the students take a simple online learning-styles test and report the results as part of their mathematical autobiography, due at the end of the first week of class.

Many students find the material in mathematics courses difficult, and taking tests very stressful. Although a certain level of stress is a useful motivator, research has shown that high levels of stress reduce performance on complex tasks. In its extreme form, this "math anxiety" can cause a student's brain to seize up during a test and prevent them from demonstrating things they know very well. I do several things to try to reduce student stress so that I can maintain high standards without damaging the students too much. My first method is to maintain a friendly classroom environment, and in general be nice to the students; this is a useful technique, but only goes so far. My second method is to maintain "no surprises." Research has shown that certainty about an event, even if that event is unpleasant, reduces stress. To increase certainty, I distribute a complete schedule for the quarter on the first day; there are no pop quizzes or other surprise events. Since tests are especially stressful, I distribute test guides with sample questions, and then use very similar questions on the test. Although my creative instincts urge me to make innovative questions, I have learned that even the standard questions are new to the student, and are a better measure of the central topics of the course. By providing sample questions, I can also increase the level of difficulty of the questions without being unfair. My third method to reduce stress is to not rely too much on tests to determine the students' grades. I balance the tests with writing assignments, as described in detail in Section IV.B.

My teaching and the department mission

At the introductory undergraduate level, the department teaches a large number of service courses. I have taught a variety of these (163A, 163B, 250, 263A, 263B, 266A, 266B), and so am able to help with this portion of the mission. At the upper undergraduate level, the department serves both its own majors and those from other departments. So far I have only taught two courses in my specialty at this level (344, 446) but would certainly be interested in teaching others. At the graduate level, I have taught the core course in my area (640ABC Numerical Analysis) and a more basic course in that area (546). These courses serve both our graduate students and those in science and engineering. Doctoral students in our department can take a comprehensive exam in this subject, based on this core course.

Another aspect of the department's teaching mission is the training of students in research. I have recently become heavily involved in this effort, through running a research group of graduate and undergraduate students. This activity is described in detail in Section IV.F.

IV.B Innovative Teaching

It sometimes seems that "innovative" teaching is valued, without important additional adjectives like "worthwhile" or "effective." I try "innovative" methods that seem reasonable, and attempt to master those that work for me. I do not consider myself a great educational innovator, but some of my methods, as described in Section IV.A, are considered unconventional within the department.

In one area, mathematical writing, I have developed an innovative approach. In 1999 I led the creation of the *Good Problems* method for integrating regular writing assignments into mathematics courses. A sample of the materials for this method are included in Section IV.E, and the full packet in given in the ancillary materials. After some initial experimentation, I have given weekly or biweekly good problems in every class since 2001, from freshman business calculus to graduate numerical analysis. I am very happy with the improvement in writing that I see over the course of the quarter. I have also had many students say that doing the *Good Problems* helped them grasp the important mathematical concepts.

Mathematical writing assignments have been recognized as a useful tool for getting students to organize their thoughts and grasp concepts. The ability to write about mathematics is also a useful life skill. Many mathematicians deride writing assignments, however, because they imagine problems like "explain how you feel about the derivative." Others incorporate writing only within large projects, and without actual instruction on how to write well. Although such projects may be a good idea for other reasons, they do little to develop writing skills — instead they mainly demonstrate the students' lack of skill. I believe the *Good Problems* method provides a solution to these problems by focusing on actual math problems and building up writing skills gradually.

The method itself is easy to use, and does not distract form the course content. Select homework problems are designated as "good problems." The student must carefully write the solution to this problem, demonstrating specific skills at presenting mathematics. A set of six skills is taught during the term, via six handouts. Each handout identifies and explains a specific skill, and provides examples of good use of this skill and examples where this skill is lacking. At any given point in the term, the student is graded only on those skills that have already been taught. These assignments slowly build up technical writing skills without too much pain, for the students or instructor.

IV.C Courses Taught

The courses I have taught here are given in Table 1. I was the sole instructor for these courses, with no teaching assistants. Courses indicated as "off-load" were taught on a voluntary basis, often because enrollment was too low for the course to run. Managing MATH 598 Internship involves only communicating with the student's employer to verify that they completed the internship. Prior to coming here, I taught courses in Calculus, Numerical Analysis, and Fourier Series.

IV.D Interdisciplinary Teaching

I have not participated in any formal interdisciplinary teaching. I have taught MATH 266A,B *Calculus with Applications to Biology*, which specifically serves a biology audience, and MATH 344 *Numerical Methods for Civil and Mechanical Engineers*, which serves an engineering audience. MATH 640A,B,C *Numerical Analysis* often draws engineering graduate students.

Table 1: Courses I have taught at Ohio University.

Term	MATH	Title	Students	Notes
20081	692	Project in Computational Mathematics	1	off-load
20081	263A	Calculus	39	
20081	163A	Introduction to Calculus	36	
20074	598	Internship	1	off-load
20073	598	Internship	1	off-load
20073	344	Numerical Methods for Civil and Mechanical Engineers	28	
20072	344	Numerical Methods for Civil and Mechanical Engineers	18	
20072	163A	Introduction to Calculus	43	
20072	598	Internship	1	off-load
20071	344	Numerical Methods for Civil and Mechanical Engineers	17	
20071	163A	Introduction to Calculus	37	
20071	598	Internship	1	off-load
20064	598	Internship	1	off-load
20063	266B	Calculus with Applications to Biology II	15	
20063	640C	Numerical Analysis	2	off-load
20063	690	Independent Study	1	off-load
20063	598	Internship	1	off-load
20062	250	Introduction to Probability and Statistics I	40	
20062	266B	Calculus with Applications to Biology II	20	
20062	640B	Numerical Analysis	2	off-load
20062	690	Independent Study	1	off-load
20062	692	Project in Computational Mathematics	1	off-load
20061	266A	Calculus with Applications to Biology I	32	
20061	640A	Numerical Analysis	4	
20061	598	Internship	1	off-load
20054	598	Internship	4	off-load
20053	266B	Calculus with Applications to Biology II	18	
20053	4/546	Numerical Linear Algebra	4	
20052	266A	Calculus with Applications to Biology I	42	
20052	163A	Introduction to Calculus	41	
20043	640C	Numerical Analysis	4	off-load
20043	250	Introduction to Probability and Statistics I	43	
20042	640B	Numerical Analysis	6	
20041	640A	Numerical Analysis	13	
20041	163A	Introduction to Calculus	40	
20033	640C	Numerical Analysis	4	off-load
20033	263B	Calculus	31	
20032	163B	Introduction to Calculus (part 2)	20	
20032	640B	Numerical Analysis	4	
20031	640A	Numerical Analysis	5	

IV.E Evidence of Effectiveness

The items on the mathematics department student evaluation form are given in Table 2. The summary of my ratings is given the Table 3, along with those of all instructors of the same course that term, when available. Copies of all the actual written student comments for Math 163A *Introduction to Calculus* in the winter of 2007 are included in the ancillary materials.

The remainder of this section contains:

1. Peer evaluations, including a summary evaluation by Dr. Todd Young.
2. An unsolicited letter from Dr. James E. Schultz, formerly the Robert L. Morton Professor of Mathematics Education.
3. A sample of the *Good Problems* materials mentioned in Section IV.B, which I use in all my classes. The complete packet is included in the ancillary materials.
4. The syllabus and a sample of the tests and other materials from Math 163A *Introduction to Calculus* in the winter of 2007. The complete set of set of materials for this course is included in the ancillary materials. These materials are representative of my materials for a standard undergraduate course. (Materials from other courses are available on my webpage.)
5. The syllabus and a sample of the tests and other materials from Math 640A *Numerical Analysis* from the fall of 2005, as an example of the (less-structured) materials for an advanced graduate course. The complete set of set of materials for this course is included in the ancillary materials.

Table 2: The items on the mathematics department student evaluation form. Items are grouped into three sections. Students rate from 1 to 5, with 1 being "agree strongly" and 5 being "strongly disagree".

Instructional Delivery: Evaluates how well the instructor presented information to the students.

item #	Statement
1	The instructor presented material in an effective manner that contributed to the mastery of the course content.
2	The instructor demonstrated a genuine interest in educating students.
3	The instructor was courteous to all students.
4	Examples, illustration, or demonstration made by the instructor helped clarify the material.
5	The instructor appeared to have thorough knowledge of the subject matter.
6	Participation was encouraged by instructor.
7	The instructor helped increase my appreciation for, and knowledge and skill in, the subject matter.

Instructional Design: Evaluates the structure of the class, including resources, assignments, and evaluation procedures.

item #	Statement
8	Grading criteria was understandable.
9	Feedback from the instructor was constructive.
10	Assignments seemed relevant to course content.
11	Exams, quizzes, projects, or other methods of evaluation fairly evaluated the students.
12	Textbooks, lab materials, readings, and resource material were of instructive value.

Course Management: Evaluates how the instructor conducted procedure pertinent to the class.

item #	Statement
13	Followed the syllabus.
14	Was well organized.
15	Maintained an environment conducive to learning.
16	Kept students informed of their progress.
17	Was available during posted office hours.

Table 3: A summary of my ratings on the student evaluations. # = Number of responses. Overall is the mean of all items. Del. = "Instructional Delivery" is the mean of items 1–7. Des. = "Instructional Design" is the mean of items 8–12. Man. = "Course Management" is the mean of items 13–17.

		Me					All instructors of this course				
Term	MATH	#	Overall	Del.	Des.	Man.	#	Overall	Del.	Des.	Man.
20081	263A	22	1.46	1.42	1.62	1.36	120	1.72	1.72	1.76	1.68
20081	163A	21	1.54	1.60	1.70	1.30	350	1.73	1.79	1.73	1.64
20073	344	23	1.83	1.98	1.87	1.59					
20072	344	14	1.93	2.04	2.27	1.44					
20072	163A	25	1.84	1.92	1.86	1.70	238	1.65	1.74	1.65	1.53
20071	344	14	1.98	2.03	1.96	1.93					
20071	163A	28	1.84	1.95	1.90	1.64	308	2.11	2.19	2.14	1.97
20063	266B	10	1.51	1.49	1.50	1.56					
20063	640C	2	1.12	1.00	1.40	1.00					
20062	250	36	1.99	2.04	2.08	1.82	175	2.00	2.02	2.09	1.88
20062	266B	16	1.51	1.48	1.80	1.25					
20062	640B	2	1.03	1.00	1.10	1.00					
20062	692	1	1.00	1.00	1.00	1.00					
20061	266A	24	1.87	1.90	2.02	1.67					
20061	640A	3	1.43	1.67	1.40	1.13					
20053	266B	15	1.49	1.48	1.68	1.32	44	2.11	2.23	2.09	1.98
20053	4/546	1	1.06	1.14	1.00	1.00					
20052	266A	24	1.99	2.07	2.01	1.86	37	2.29	2.35	2.16	2.33
20052	163A	28	2.03	2.21	1.88	1.95	278	1.99	2.06	2.00	1.88
20043	640C	4	1.34	1.21	1.55	1.30					
20043	250	26	1.95	2.01	2.03	1.79	213	2.05	2.10	2.11	1.93
20042	640B	6	1.12	1.10	1.13	1.14					
20041	640A	11	1.29	1.25	1.31	1.33					
20041	163A	14	1.97	1.94	2.01	1.96	227	2.14	2.24	2.10	2.02
20033	640C	4	1.68	1.68	1.75	1.60					
20033	263B	23	2.17	2.25	2.33	1.89	146	2.17	2.20	2.25	2.05
20032	163B	19	2.11	2.18	2.14	2.00	27	2.09	2.27	1.95	1.98
20032	640B	3	1.39	1.43	1.40	1.33					
20031	640A	5	1.55	1.43	1.52	1.76					

OHIO UNIVERSITY

Robert L. Morton Professor of Mathematics Education
117 McCracken Hall
Ohio University
Athens, OH 45701

June 21, 2005

Dr. Martin Mohlenkamp
Department of Mathematics
Campus

Dear Martin,

I am writing this unsolicited letter to express my support for your regular involvement in the mathematics education seminar over the past several years. I strongly believe that effectively addressing the challenges facing mathematics education requires the cooperative efforts of mathematicians, mathematics educators, and classroom teachers. Your involvement in the seminar is a significant contribution to the University and mathematics education community.

Your supportive yet critical participation as a mathematician in the discussions provides an important perspective. I encourage you to continue and urge the Department of Mathematics and College of Arts and Sciences to recognize this significant contribution when evaluating your work.

Cordially,

James E. Schultz

c. Dr. Jeff Connor

Student's Guide to Good Problems

Many students believe that writing has nothing to do with mathematics. **This is false.** In any field, it is important to be able to communicate your ideas and results to others. Exactly how you communicate them varies from field to field, but in nearly all fields it is through writing. Mathematical writing, however, has its own particular style. The emphasis is on clarity and precision, and not on the clever turn of phrase.

The "Good Problems" program is designed to teach you how to write about mathematics coherently. It provides you a set of specific writing skills and gives you enough practice through regular assignments so that writing well will become a habit. It is also designed to be as painless as possible.

Throughout the term, some homework problems will be designated "good problems". You need to write the solution to this problem carefully, in good "presentation" format. It will be graded partly for correctness, but with emphasis on presentation.

During the term you will receive six handouts, each addressing a particular writing skill. The good problem is graded only on those skills already covered, with greater emphasis on the newest skill. By the end of the term you will have acquired the basic skills of mathematical writing. The handouts are:

- Laying out the Problem.
- Flow.
- Mathematical Symbols.
- Logical Connectives.
- Graphs.
- Introductions and Conclusions.

Each handout consists of a description of a particular writing skill, motivation for the importance of this skill, whatever explicit rules can be given, and examples of good and bad presentation emphasizing that skill. We use a consistent format for the examples.

Bad: *An example of poor writing looks like this.*

Good: An example of good writing looks like this.

A comment within an example looks like this.

If you already have these basic writing skills, then it should only take you ten extra minutes to write up a good problem. If you do not have these skills, then you will have to spend extra effort in the short term to acquire them. In the long term, however, these skills will save you much time and effort. Besides making future writing assignments easier, these skills will help you with problem-solving (by encouraging organization and logic) and with reading mathematics. These skills should be carried into future mathematics courses, and become a useful life skill.

Laying out the Problem

Presentation is very important in mathematics, just as it is in other fields. The main reason for this is clarity. Good presentation allows you to communicate more clearly the content of your work. A secondary reason is the appearance of competence. Careful presentation makes the reader think you were also careful with the content of your work. Although good presentation is rarely successful at bluffing past poor content, poor presentation can easily ruin good content.

The first thing the reader evaluates is the overall visual layout of the problem. Exactly how polished this should be will depend on the purpose of your writing. For our purposes, we have the following rules:

- Put the problem on its own sheet of paper. Regular quality paper is fine, but there should be no ragged edges or tears.
- If you need more than one page, staple (no folded corners!) them together and put your name and the page number on each page.
- Leave margins on all sides.
- Print neatly or type. Do not switch colors or from pen to pencil in the middle of the problem. (You can use different colors to highlight if you wish.)
- If you had to cross out material or erased a lot and left smudges, rewrite the problem. (It is a good habit to solve the problem first on scrap paper and then copy it neatly.)

The reader next needs to know what it is they are reading. On top of the first page, put:

1. your full name,
2. the course and section or recitation number, and
3. the assignment number and/or the date the assignment is due.

Next is the format for the problem itself:

- Label the problem with the chapter, section, and problem number.
- Write out the entire question, including any instructions. If the question refers to another problem, include the relevant information from that problem. The goal is to make your work as self-contained as possible, so the reader does not need to look anything up.
- Do the problem in some logical order. Do not do the problem in several disjoint pieces connected by arrows.

All these rules may seem picky. Once you have learned this way to lay out your work, you will understand the principles behind these rules. Then you will know when to change the rules.

Acknowledgements: If you had help with this problem, or in any way include work that is not your own, then give proper credit. Not only is this the nice thing to do, but also it will protect you from allegations of cheating.

Good presentation is important in mathematics. It lets you communicate your work more clearly and lends it credibility.

Laying out the Problem, page 2. Good Problems: November 13, 2004

Example:

Good:

Full Name
Course and section or recitation number
Assignment number
Date

Section number, problem number. Full text of the problem . . . blah . . . blah . . . blah . . . blah.

In this problem we . . . blah . . . blah . . . blah . . . blah . . . blah . . . blah . . . blah . . . blah.
There is a separate handout on introductions.

Since . . . blah . . . blah . . . we know . . . blah . . . blah.
There is a separate handout on logical connectives.

$\Longrightarrow$ blah
$\Longrightarrow$ blah blah
$\therefore$ blah!

There is a separate handout on mathematical symbols.

From this we can conclude . . . blah . . . blah . . . blah . . . blah and so $x = 3\pi$ . . . . . . blah.
There is a separate handout on incorporating mathematics into sentences and other issues of 'flow'.

By graphing our solution, we can see that . . . blah.

There is a separate handout on graphing.

By using . . . blah . . . blah . . . we were able to solve this problem . . . blah . . . blah . . . blah . . . blah.
There is a separate handout on conclusions.

Acknowledgements: I worked with so-and-so on this problem and also had help from so-and-so.

Flow

Written mathematics must be readable. This may seem trivial, but it is an important point. You should be able to read your work aloud to a classmate and have them understand your solution. If you need to add any explanations, these should be included in your written work. The most common mistake is to write mathematics without using enough words. All writing, even mathematics, should consist of complete sentences. These should explain the problem by providing both the method and justification for each step of the solution.

Why are sentences important in mathematics?

Although sometimes it seems hard to read textbooks, it would be much harder to understand if they only had equations and no sentences. The situation is similar in lecture: if the professor just listed formulas on the chalkboard without talking about them, leaving you to figure out what was being done in each step, how much could you understand from the lecture? Neither of these would be a good way for most students to learn, since sentences are necessary to explain the mathematics.

Why should students use sentences in a Mathematics class?

In a Mathematics class, you should explain homework solutions using complete sentences. That means linking together thoughts with words and embedding equations into sentences. Going through the extra work to do this will benefit you in several ways:

- Writing down your thoughts and organizing them into complete sentences will help you to understand the method of solution better.
- When you look back on homework to study for a test, or later on in another class, you will understand what you were doing on each problem and the mathematics behind it.
- Other people (teacher, classmates, grader,...) will understand what you are doing at each step, and why you are doing it. This way, you won't lose points for skipping steps or solving the problem in an unusual way.
- Communicating your work will be essential in whatever field you choose. Even though the fields are stereotypically weak on writing, engineers and scientists spend a surprising amount of time writing reports and giving oral presentations.

The Royal "We".

It is customary to write mathematics using "we" instead of "I" and "you". Think of yourself as the tour guide, showing the sights to your reader. You and the reader together make "we".

Your Audience

Keep in mind who your audience is. For the Good Problems, pretend that you are writing for a classmate who has not seen this problem before. Write with enough detail so that they can follow your explanations. A good way to test if you have written enough is to read your work aloud to another person. If you need to add any words to make it make sense, then these words should be included in your written work.

Examples:

- Find an equation for the tangent to the curve $y = x + \frac{2}{x}$ at the point $(1, 3)$.

  Good:

  We first check that $(1, 3)$ is a point on the curve by plugging these values in: $3 = 1 + 2/1$. The derivative of the curve $y = x + \frac{2}{x}$ is

  $$\frac{dy}{dx} = 1 - \frac{2}{x^2}. \tag{1}$$

  A line tangent to the curve at the point $(1, 3)$ will have slope

  $$\left.\frac{dy}{dx}\right|_{x=1} = 1 - \frac{2}{(-1)^2} = -1.$$

  Using the point-slope formula with $m = -1$, $x_0 = 1$, and $y_0 = 3$ gives the formula for the line $y - 3 = -1(x - 1)$. Solving for y and simplifying gives

  $$y = -x + 4.$$

  This is the equation of the line tangent to the curve at that point.

- **Bad:** *We use calculus to find that* $y = 3x^2 + 1$ *has a slope of* 3 *at* $x = 1/2$.

  Good: To find the slope of the curve $y = 3x^2 + 1$ at the point $x = 1/2$ we find $\frac{dy}{dx}$ evaluated at $x = 1/2$. We compute

  $$\begin{aligned} \frac{dy}{dx} &= 6x, \\ \Rightarrow \left.\frac{dy}{dx}\right|_{x=1/2} &= 6\left(\frac{1}{2}\right) = 3. \end{aligned}$$

- **Bad:**

  $$\begin{aligned} \frac{d}{dx}\left((x^2+1)(x^3+3)\right) &= (x^2+1)3x^2 + (x^3+3)2x \\ &= 5x^4 + 3x^2 + 6x \end{aligned}$$

  is the derivative.

  Good: To take the derivative of a product of 2 functions, we use the product rule, $(fg)' = f'g + fg'$. In our case we have

  $$\begin{aligned} \frac{d}{dx}\left((x^2+1)(x^3+3)\right) &= \left(\frac{d}{dx}(x^2+1)\right)(x^3+3) + (x^2+1)\left(\frac{d}{dx}(x^3+3)\right) \\ &= (2x)(x^3+3) + (x^2+1)(3x^2) \\ &= 5x^4 + 3x^2 + 6x. \end{aligned}$$

- You may have noticed that one of the equations above has been labeled equation number one by putting "(1)" at the right hand margin. If you need to refer back to an equation or figure, label it and then refer to it by its label. Do not draw arrows.

  Bad: *Using the equation from before, the slope is* -1. Which equation?

  Good: Using (1), the derivative at $x = 1$ is -1.

Logical Connectives

Good Problems: November 13, 2004

Mathematics has its own language. As with any language, effective communication depends on logically connecting components. Even the simplest "real" mathematical problems require at least a small amount of reasoning, so it is very important that you develop a feeling for *formal (mathematical) logic*.

Consider, for example, the two sentences "There are 10 people waiting for the bus" and "The bus is late." What, if anything, is the logical connection between these two sentences? Does one *logically imply* the other? Similarly, the two mathematical statements "$r^2 + r - 2 = 0$" and "$r = 1$ or $r = -2$" need to be connected, otherwise they are merely two random statements that convey no useful information. Warning: when mathematicians talk about implication, it means that one thing **must** be true as a consequence of another; not that it can be true, or might be true sometimes.

Words and symbols that tie statements together logically are called *logical connectives*. They allow you to communicate the reasoning that has led you to your conclusion. Possibly the most important of these is *implication* — the idea that the next statement is a logical consequence of the previous one. This concept can be conveyed by the use of words such as: therefore, hence, and so, thus, since, if … then …, this implies, etc. In the middle of mathematical calculations, we can represent these by the implication symbol ($\Rightarrow$). For example

$$x + 7y^2 = 3 \quad \Rightarrow \quad y = \pm\sqrt{\frac{3-x}{7}}; \tag{1}$$

$$x \in (0, \infty) \quad \Rightarrow \quad \cos(x) \in [-1, 1]. \tag{2}$$

Converse

Note that "statement A $\Rightarrow$ statement B" does **not** necessarily mean that the *logical converse* — "statement B $\Rightarrow$ statement A" — is also true. Logical implication is a matter of cause and effect; the logical converse is simply the reverse cause-effect situation (which may not be true). Consider the following, everyday example:

- "I am running to class because I am late."

It should be clear that the logical converse of these is not true:

- "I am late to class because I am running."

For examples (1) and (2) above:

$$x + 7y^2 = 3 \quad \Leftarrow \quad y = \pm\sqrt{\frac{3-x}{7}};$$

$$x \in (0, \infty) \quad \not\Leftarrow \quad \cos(x) \in [-1, 1]. \quad \boxed{\text{Since } x < 0 \Rightarrow \cos(x) \in [-1, 1] \text{ also.}}$$

Contrapositive

We have seen that simply reversing a logical statement can lead to problems. There is a way, however, to invert an implication so that the inverted statement is also true. This is known as the *contrapositive*. Consider the statement "$A \Rightarrow B$"; this means "if A is true, then B is true". The contrapositive is "if B is **false**, then A must be false".

Here is an example of how these concepts fit together:

- If X is a cat, it is an animal. [Accept as true]
- If X is an animal, it is a cat. [FALSE (converse)]
- If X is not an animal, it is not a cat. [TRUE (contrapositive)]
- If X is not a cat, it is not an animal.
  [FALSE. This is the contrapositive of the second statement, which is false.]

Equivalence

When the implication works both ways, we say that the two statements are *equivalent* and we may use the equivalence symbol (⇔); in words we may say "A is equivalent to B" or "A if and only if B". If two statements are equivalent, we may use any of the implication symbols (⇒, ⇐, or ⇔). Which connective we use depends on what we are trying to show. In (1) above, if we are trying to obtain a formula for y, we would probably just use "⇒", even though the stronger statement ("⇔") is also true. If, however, we wanted to show that two statements about x and y were equivalent, we would write $x + 7y^2 = 3 \Leftrightarrow y = \pm\sqrt{\frac{3-x}{7}}$.

Careful logic is the heart and soul of mathematics; learn to reason with watertight arguments and use logical connectives to explain your reasoning processes.

Example:

What is the greatest amount of water that a right-cylindrical water tank can hold if there is $100m^2$ of material from which to construct it?

Good: Let the height of the tank be h and the radius be r. Then the volume of the tank is $V(h, r) = \pi hr^2$ and the surface area is $S(h, r) = 2\pi r^2 + 2\pi rh = 2\pi r(r + h)$. Since we require $V > 0$, we can assume that $r, h > 0$. If we have $100m^2$ of material, then $S = 100$, and so

$$\begin{aligned} S &= 2\pi r(r+h) = 100 \\ &\Rightarrow r + h = \frac{100}{2\pi r} \\ &\Rightarrow h = \frac{50}{\pi r} - r \\ &\Rightarrow V = \pi hr^2 = 50r - \pi r^3. \end{aligned}$$

To find the maximum volume, we look for r such that $\frac{dV}{dr} = 0$. Differentiating with respect to r gives $\frac{dV}{dr} = 50 - 3\pi r^2$. Hence,

$$\begin{aligned} \frac{dV}{dr} = 0 &\Leftrightarrow 50 - 3\pi r^2 = 0 \\ &\Leftrightarrow r^2 = \frac{50}{3\pi} \\ &\Leftrightarrow r = \pm\sqrt{\frac{50}{3\pi}}. \end{aligned}$$

Since $r > 0$, we can use the second derivative test and find $\frac{d^2V}{dr^2} = -6\pi r < 0$. This implies that $r = \sqrt{\frac{50}{3\pi}} \approx 2.3033$ maximizes V. From above, the optimal volume is

$$\begin{aligned} V &= 50r - \pi r^3 \\ &= \left(50 - \pi\left(\frac{50}{3\pi}\right)\right)\sqrt{\frac{50}{3\pi}} \\ &= \left(\frac{100}{3}\right)\sqrt{\frac{50}{3\pi}} \approx 76.7765. \end{aligned}$$

Thus we have found that the greatest amount of water such a tank can hold is (approximately) $76.7765m^3$.

OHIO UNIVERSITY **Department of Mathematics**

MATH 163A A07(04894), Winter 2007

Introduction to Calculus

Catalog Description:
: Presents a survey of basic concepts of calculus. For students who want an introduction to calculus, but do not need the depth of 263A-B-C. Note: Students cannot earn credit for both 163A and either of 263A of 266A.

Prerequisites:
: MATH 113 or Placement level 2 or higher.

Instructor:
: Martin J. Mohlenkamp, mjm@math.ohiou.edu, (740)593-1259, 315-B Morton Hall.
: Office hours: Monday 2-3pm, Tuesday 10-11am, Thursday 5-6pm, and Friday 10-11am.

Web page:
: http://www.math.ohiou.edu/~mjm/20072/163A.

Class hours/ location:
: MTuThF 12:10-1pm in 218 Morton Hall.

Text:
: **Calculus with Applications (brief version)**, eighth edition, by Margaret L. Lial, Raymond N. Greenwell, and Nathan P. Ritchey; Addison Wesley, 2005.

Homework:
: Several problems from each section of the book are assigned. These problems will not be collected or graded, but you will need to do them in order to learn.

Good Problems:
: Six Good Problems are assigned, and will be collected and graded. These are homework problems that will be graded half on content and half on presentation. The idea is to practice writing mathematics regularly but in small pieces.

Tests:
: There will be four mid-term tests, in class. Calculators are not permitted.

Final Exam:
: The final exam is on Wednesday, March 14, at 2:30 pm in our regular classroom. Calculators are not permitted.

Grade:
: Each Good Problem is worth 1 unit, each test is worth 2 units, and the final is worth 4 units. Your lowest 2 units will be dropped and then your average is computed and a 90% guarantees you at least an A-, 80% a B-, 70% a C-, and 60% a D-.

Missed or Late work:
: Only reasons given in advance of a missed test will be considered; otherwise a score of 0 will be given. Late Good Problems are penalized 5% for each 24 hour period or part thereof, excluding weekends and holidays. You can resubmit good problems to improve your score, but the late penalty will apply.

Attendance:
: Attendance is assumed but is not counted in your grade. It is your responsibility to find out any announcements made in class.

Academic Dishonesty:
: You are strongly encouraged to work together on the homework. You can work together on the Good Problems, but you must acknowledge in writing what help you received and from whom. The tests and final exam must be your own work, and without the aid of notes, etc. Dishonesty will result in a zero on that work, and possible failure in the class and a report to the university judiciaries.

Supplemental Instruction:
: SI "provides free, out-of-class study sessions led by an Ohio University undergraduate student who has already taken the course. Used throughout the U.S. and the world, the SI program has proven highly successful in increasing student achievement and retention." Check with the academic advancement center for more information and the schedule.

Special Needs:
: If you have specific physical, psychiatric, or learning disabilities and require accommodations, please let me know as soon as possible so that your learning needs may be appropriately met.

Learning Resources:

- Your classmates are your best resource. Use them!
- The Academic Advancement Center's Math Center http://www.ohiou.edu/aac/math has drop-in help, tutors, online help, and a telephone hotline.
- The calculus page http://www.math.ucdavis.edu/~calculus/ at UC Davis has links to many Calculus resources.

Schedule

The Good Problems and Tests are fixed, but we may not cover sections on exactly the days shown.

Week	Date	Section	Homework/Materials (ungraded)	Good Problem/ Test
1	January 4	Introduction; 1.1	1-37odd,39-42,45-59odd	
	January 5	1.2	1-6,7-15odd,16,17	
		2.1	1-8,9-65odd	
2	January 8	2.2	1-7,9-19odd,20-28,29-41odd	Good Problem 1: Mathematical Autobiography, using Layout
	January 9	2.3	1-3,7-26,27-39odd	
	January 11	Review		
	January 12		study guide	Test 1 on 1.1,1.2,2.1-2.3
3	January 15	*Martin Luther King Jr. Day, no class*		
	January 16	3.1	1-15,17-55odd,56,61,62,73	(drop deadline January 17)
	January 18	3.2	1-6,7-13odd,19-29odd,30	
	January 19	3.3	1-15odd,21,22	Good Problem 2: Section 3.1 #42, using Flow
4	January 22	3.4	1-10,11-23odd,31-37	
	January 23	4.1	1-45odd,55	
	January 25	4.2	1-31odd,37	
	January 26			Good Problem 3: Section 3.4 #20 with a graph, using Graphs
5	January 29	4.3	1-41odd,42,43-51odd,54	
	January 30			
	February 1	Review		
	February 2		study guide	Test 2 on 3.1-3.4,4.1-4.3
6	February 5	5.1	1-8,9-23odd,29,36	
	February 6	5.2	1-8,9-23odd,29	(drop deadline with WP/WF)
	February 8			
	February 9	5.3	1-11odd,17-23odd,27-32,33-39odd,49-53odd	Good Problem 4: Section 5.1 #14, using Logic
7	February 12	5.4	3-19odd,35-39	
	February 13			
	February 15	Review		
	February 16		study guide	Test 3 on 5.1-5.4
8	February 19	6.1	1-9,11-23odd,27-31odd,35	
	February 20	6.2	1-4,7-27odd	
	February 22			
	February 23	9.1	1-15odd,17-27	Good Problem 5: Section 6.2 #12, using Intros
9	February 26	9.2	1-5odd,11,17,21-25odd,33-39odd,47	
	February 27			
	March 1	Review		
	March 2		study guide	Test 4 on 6.1,6.2,9.1,9.2
10	March 5	9.3	1-15odd,19,20,21-29odd	
	March 6			
	March 8	9.4	1-13odd,15-17,19	
	March 9	Review		Good Problem 6: Section 9.3 #14, using Symbols
11	March 14	study guide	**Final Exam** Wednesday, at 2:30 pm, in our classroom	

Math 163A A07 **Winter 2007**

Your first Good Problem is due in class on Monday January 8. Good Problems are graded half on content and half on presentation. The only specific presentation skills that you are responsible for so far are described in the *Layout* handout. The target length is 2 pages.

To determine your preferred learning style, take the online test at `http://iliad.cats.ohiou.edu/vark/questionnaire.htm` or `http://www.vark-learn.com/english/page.asp?p=questionnaire`. (Note that on this test you can choose more than one answer to each question.)

Problem: Write your mathematical autobiography. Include

- your background,
- your current interests and future goals,
- why you are taking this class,
- your preferred learning style(s), and
- whatever else is relevant or interesting.

Math 163A **Guide for Test 2**

Here are some sample questions from old tests. Some topics that we covered are not represented by these questions, but are still fair game.

1. Compute the following limits:
   (a) $\lim_{x\to 2} \frac{x-2}{x^2-5x+6}$
   (b) $\lim_{h\to 0} \frac{x^2-(x-2h)^2}{h}$
   (c) $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$
   (d) $\lim_{x\to -\infty} \frac{3x^3-4}{2x^3-2}$
2. Let $f(x) = -x^2+3$.
   (a) Using the definition of the derivative as a limit, compute $f'(x)$.
   (b) Find the equation for the tangent line at $x = 2$.
   (c) Graph $f(x)$ and the tangent line.
3. Compute the following derivatives:
   (a) $f(x) = 2 + x + \frac{3}{x} - \sqrt{x} - 5x^7 + x^{3/4}$
   $\Rightarrow f'(x) =$
   (b) $y = \frac{x^3+x}{x} \Rightarrow \frac{dy}{dx} =$
   (c) $D_x\left[(x^9+x^8+x^5+3)(1+2x^2+9x^3-4x^4)\right] =$
   (d) $\frac{d}{dx}\left[(x^9+2x^{1/3}+x^5+3)^4\right] =$
4. Compute the following derivatives:
   (a) If $f(x) = \frac{u(x)v(x)}{w(x)}$, then in terms of $u(x)$, $v(x)$, $w(x)$, $u'(x)$, $v'(x)$, and $w'(x)$, we have $f'(x) =$
   (b) $y = (3+x^4)^8x^3 \Rightarrow \frac{dy}{dx} =$
   (c) $D_x\left[\left((x^9+x^8+x^5+3)(1+2x^2+x^3-4x^4)\right)^9\right] =$

Math 163A-A07 *Winter 2007* *Test 2* *p.1*

Name: ______________________________

#	possible	score
1	24	
2	16	
3	28	
4	32	
	100	

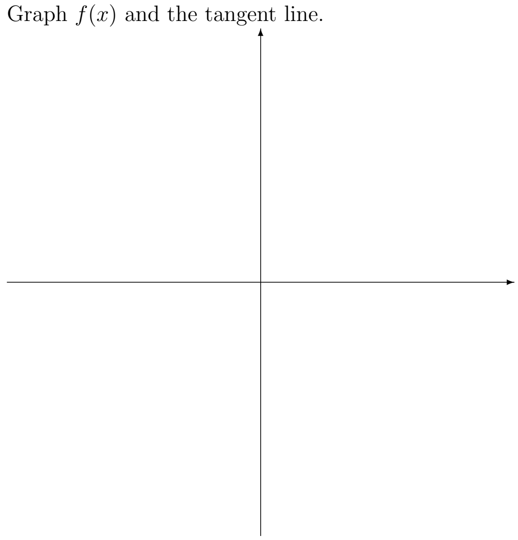
No books, notes, or calculators are allowed.
Show your work and give reasons for your conclusions.
Use the back of the sheet if you need more space.

1. Let $f(x) = x^2 - 3$.

   (a) Using the definition of the derivative as a limit, compute $f'(x)$.

   (b) Find the equation for the tangent line at $x = -2$.

   (c) Graph $f(x)$ and the tangent line.



Math 163A-A07 *Winter 2007* *Test 2* *p.2*

2. Compute the following limits:

   (a) $\lim_{x \to 4} \frac{x-4}{\sqrt{x}-2}$

   (b) $\lim_{x \to \infty} \frac{1+x+5x^2}{2x^2-3}$

3. Compute the following derivatives:

   (a) $D_x[x-6+3x^2+\sqrt{x}] =$

   (b) $D_x\left[x^{3/5}+\frac{-2}{x}+x^{-2}\right] =$

   (c) $D_x[(x^2+x)^{57}] =$

   (d) $D_x\left[\frac{x^3+2x+1}{x^2+x}\right] =$

Math 163A-A07 *Winter 2007* *Test 2* *p.3*

4. Compute the following derivatives:

   (a) $D_x\left[(x^9+x^8+x^5+3)(1+2x^2+9x^3-4x^4)\right] =$

   (b) $y = x^5(3+x^4)^8 \Rightarrow \dfrac{dy}{dx} =$

   (c) $D_x\left[\left(\dfrac{x^3+2x+1}{x^2+x}\right)^9\right] =$

   (d) If $f(x) = \dfrac{u(x)}{v(x)w(x)}$, then in terms of $u(x)$, $v(x)$, $w(x)$, $u'(x)$, $v'(x)$, and $w'(x)$, we have $f'(x) =$

Math 163A-A07 *Winter 2007* *Final Exam* *p.1*

Name: ______________________________

#	possible	score
1	10	
2	10	
3	20	
4	20	
5	20	
6	20	
	100	

No books, notes, or calculators are allowed.
Show your work and give reasons for your conclusions.
Use the back of the sheet if you need more space.

1. Let $f(x) = \sqrt{x}$.

   (a) Using the definition of the derivative as a limit, compute $f'(x)$.

   (b) Find the equation for the tangent line at $x = 4$.

2. Compute the following limits:

   (a) $\lim_{x \to 2} \frac{x-2}{x^2 - 5x + 6} =$

   (b) $\lim_{x \to \infty} \frac{1 + x + x^2}{2x - 3} =$

3. Compute the following derivatives:

   (a) $D_x[(x^2 + x)^{50}] =$

   (b) $D_x \left[\dfrac{x^3 + 2x + 1}{x^2 + x} \right] =$

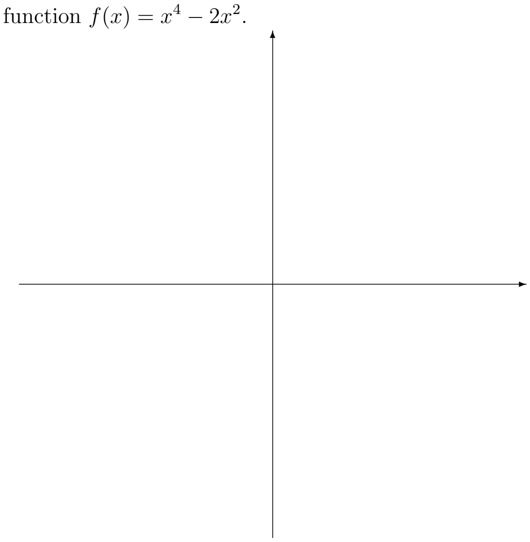
   (c) $D_x \left[(x^9 + x^8 + x^5 + 3)(1 + 2x^2 + 9x^3 - 4x^4) \right] =$

   (d) $y = (3 + x^4)^8 x^5 \Rightarrow \dfrac{dy}{dx} =$

4. You figure you can spend 24 cents on the can for your new energy drink. The material for the top is 6 cents per square inch, the material for the bottom is 4 cents per square inch, and the material for the side is 2 cents per square inch. What is the maximum volume that you could make the can? (Note: Your answer may contain ugly expressions like $\sqrt{7\pi}$; don't panic.)

Math 163A-A07 *Winter 2007* *Final Exam* *p.4*

5. Analyze and graph the function $f(x) = x^4 - 2x^2$.



6. Let $f(x, y) = 3x^2 + 7y^3 - 42xy$. Find all extrema of f and identify any saddle points.

OHIO UNIVERSITY **Department of Mathematics**

MATH 640A (04708), Fall 2005

Numerical Analysis

Catalog Description (updated this fall):
In-depth treatment of numerical aspects of linear algebra and nonlinear systems.

Contents:
In 640A we will start at the beginning of numerical analysis with some preliminaries (chapter 1), and computer arithmetic (chapter 2). We will then cover nonlinear equations (chapter 3), and linear algebra (chapters 4 and 5).

Prerequisites:
Linear Algebra (MATH 511), Advanced Calculus (MATH 560A), and [Numerical Analysis (MATH 544) or Numerical Linear Algebra (MATH 546)]. If you are interested in taking this course but may be lacking a prerequisite, please consult with me first. Less-prepared students should consider Math 544/545/546, which offers similar material.

Instructor:
Martin J. Mohlenkamp, mjm@math.ohiou.edu, (740)593-1283, 554 Morton Hall.
Office hours: Monday 2-3pm, Tuesday 10-11am, Thursday 10-11am and 5-6pm, and Friday 10-11am.

Web page:
http://www.math.ohiou.edu/~mjm/20061/640A.

Class hours/ location:
MTu(W)ThF 11:10am-12 noon in 313 Morton Hall.

Text:
Numerical Analysis: Mathematics of Scientific Computing, 3rd edition, by David Kincaid and Ward Cheney, Brooks/Cole, 2002.

Homework:
There will be weekly homework assignments, consisting of:

- Traditional paper-and-pencil problems;
- Programming problems (I support Matlab, C, and Python, but other languages are acceptable); and
- Good Problems, which are graded half on content and half on presentation.

Exams:
There will be one midterm exam, probably in the fifth week. The final exam is on Saturday, November 19, at 10:10 am in our regular classroom. Calculators are permitted for arithmetic.

Project:
There will be a medium-sized project. I will help you to choose a topic that applies numerical analysis techniques to something that you encounter in your work, in your other classes, or in your outside interests. It should be interesting and perhaps even useful. You will write a report and give a presentation in class.

Grade:
Based on homeworks 40%, midterm exam 20%, project 20%, and the final exam 20%. An average of 90% guarantees you at least an A-, 80% a B-, 70% a C-, and 60% a D-. Grades are not the point.

Missed or Late work:
Late homeworks are penalized 5% for each 24 hour period or part thereof, excluding weekends and holidays. You can resubmit a homework to improve your score, but the late penalty will apply.

Attendance:
Attendance is assumed but is not counted in your grade. It is your responsibility to find out any announcements made in class.

Academic Dishonesty:
You are strongly encouraged to work together on the homework, but you must acknowledge in writing what help you received and from whom. The project must be primarily your own work; if you receive any help then you must acknowledge in writing what help you received and from whom. The midterm and final exam must be your own work, and without the aid of notes. Dishonesty will result in a zero on that work, and possible failure in the class and a report to the university judiciaries.

Special Needs:
If you have specific physical, psychiatric, or learning disabilities and require accommodations, please let me know as soon as possible so that your learning needs may be appropriately met.

Resources:

- Your classmates are your best resource. Use them!
- LaTeX: LaTeX help 1.1; A sample LaTeX file, samplelatex (.tex, .dvi, .ps, .pdf); A sample of latex with figures incorporated: latexfig.tex and the sample figure lfig.eps.
- MATLAB: Ohio University Matlab Central; Local Matlab Quick Reference and Survival Guide;

Schedule

Subject to change.

Week	Date	Section	Homework/ Test/ etc.
1	September 6	Introduction	
	September 8	1.1	
	September 9	1.2	
2	September 12	1.3	
	September 13		Homework 1 due; using Layout
	September 15	2.1	
	September 16	2.2	
3	September 19	2.3	
	September 20		(drop deadline)
	September 22	3.1	Homework 2 due; using Flow
	September 23	3.2	
4	September 26	3.3	
	September 27		Homework 3 due; using Logic
	September 29	3.4	
	September 30	3.5	
5	October 3	3.6	
	October 4		Homework 4 due; using Symbols
	October 6	Review	
	October 7		Test on Chapters 1-3; study guide
6	October 10	4.0,5.0	(drop deadline with WP/WF)
	October 11	4.1	
	October 13	4.2	Project topic due
	October 14		
7	October 17	4.3	
	October 18	4.4	Homework 5 due; using Graphs
	October 20	4.5	
	October 21		
8	October 24	4.6	
	October 25	4.7	Homework 6 due; using Intros
	October 27		
	October 28	4.8	
9	October 31	5.1	Homework 7 due
	November 1	5.2	Project report draft due
	November 3		
	November 4	5.3	
10	November 7		
	November 8	5.4	Homework 8 due
	November 10	5.5	
	November 11		
11	November 14		Project presentations
	November 15		Project presentations; Project final report due
	November 19	**Final Exam** 10:10am-12:10pm Saturday in our classroom	

Martin J. Mohlenkamp
Last modified: Tue Aug 30 13:45:42 EDT 2005

Math 640A **Fall 2005**
Homework 1, due Tuesday 13 September.

1. Buy/ find/ acquire the textbook: *Numerical Analysis: Mathematics of Scientific Computing*, 3rd edition, by David Kincaid and Ward Cheney, Brooks/Cole, 2002. Read the book!

2. (40 points) Do this problem as a Good Problem, paying attention to the *Layout* handout. You are encouraged but not required to LaTeX your good problems. See the back of this sheet for a description of the Good Problems.

   Write your mathematical autobiography. Include your background, current interests, future goals, why you are taking this class, and whatever else is relevant or interesting. Target length is 2 pages.

3. (30 points) Section 1.1 problems 9, 12, and 23.

4. (30 points) Section 1.2 problems 8, 14, and 40.

Math 640A **Fall 2005**
Homework 6, due Tuesday 25 October.

1. (20 points) Section 4.3 computer problem 1. Put in lots of comments and print out your code.

2. (20 points) Do this problem as a Good Problem, using the handouts that you already have.

   Section 4.4 problem 2.

3. (20 points) Section 4.4 problem 40a

4. (40 points)

   (a) Section 4.5 problem 21.

   (b) Relate this problem to Section 3.2 problems 5 and 6, and thereby construct a recursive algorithm for computing A^{-1}.

   (c) Verify the product formula

   $$(I - A)^{-1} = \prod_{k=0}^{\infty}(I + A^{2^k}).$$

   (d) Compare the standard Neumann series formula

   $$(I - A)^{-1} = \sum_{k=0}^{\infty} A^k$$

   from Section 4.5 with the product formula and recursive method above. All three can be used to construct the inverse of a matrix. Which method is most efficient? Which algorithm would you recommend?

MATH 640A **Study Guide for Midterm Exam** **Fall 2005**

When: ____________________________

Where: ______________ Morton.

What is covered: Chapters 1, 2, and 3.

General principles:

- Know the core concepts thoroughly. This is not a test of your ability to memorize peripheral details.
- Calculators are forbidden and no formulas will be provided. However, the numbers will be rigged so that you do not need a calculator, and formulas can be purchased during the test.
- You should be able to **do** any of the methods we have learned. Much of the test will be straightforward computation.
- You should be able to **state** the definitions and theorems that we have encountered. See the section notes for which ones you need to be able to prove.
- You should be able to **determine** which method is appropriate for a given situation. That means you should be able to **explain** the advantages and disadvantages of each method.

Section notes:

These notes are to help guide your study, by listing some topics you definitely need to know and some that you can ignore. They are not a list of all that you need to know.

Section 1.1 Know the theorems from Calculus and Taylor's theorem with Lagrange remainder. You can ignore Taylor's theorem with integral remainder and Taylor's theorem in two variables.

Section 1.2 Ignore the mean-value theorem for integrals and the implicit function theorem.

Section 1.3 Know the statements of the theorems and the proof for the theorem on stable difference equations, but not the proof for the theorem on null space.

Section 2.1 Know the terminology and be able to do floating-point error analysis under the assumption of no cancellation. Know the statement of the theorem on relative roundoff error in adding but not the proof.

Section 2.2 Be able to detect the loss of significance and work around it. Ignore interval arithmetic.

Section 2.3 Ignore condition number of a matrix.

Section 3.1 Know bisection thoroughly.

Section 3.2 Know Newton's method thoroughly, including when it fails, the proof for quadratic convergence, and how to do it on systems. Ignore implicit functions.

Section 3.3 Know the secant method, including error analysis.

Section 3.4 Know how to use the contractive mapping theorem but not its proof. Be able to test for contraction using the derivative. Be able to compute order of convergence.

Section 3.5 Know the theorems on algebra and localization of roots, but not their proofs. Know Horner's algorithm and how to use it with Newton's method and deflation to find all the roots. Know the theorem on successive Newton iterates but not the proof. Ignore Bairstow's method and Laguerre's method.

Section 3.6 Be able to set up the homotopy for a system with one or two variables, and describe how to use it.

MATH 640A **Midterm Exam** **Fall 2005**

Name: ______________________ Staple this sheet to your exam.
No books, notes, or calculators are allowed.
Show your work and **give reasons** for your conclusions.

1. (20 points) Bound the error of the (Taylor) approximation
$$\sqrt{65} \approx 8 + \frac{1}{16}\,.$$
2. (20 points) Let a, b, and c be positive machine numbers on a machine with unit roundoff ϵ. Estimate the relative error in computing $(ab)+(ac)$ and the relative error in computing $a(b+c)$. Which way would you compute this quantity?
3. (20 points) Give an example of a computation where cancellation causes loss of precision, and then show a method to compute the same quantity without loss of precision.
4. (20 points) Find the fixed points and rate of convergence for the iteration
$$x_{n+1} = \frac{3x_n - x_n^3}{2}\,.$$
5. (20 points) We wish to find a root of the polynomial
$$p(x) = 2x^4 - 3x^2 + 3x - 4\,.$$
   (a) Using Horner's algorithm for evaluations, compute one step of Newton's method starting with $x_0 = -2$.
   (b) Show that Newton's method is quadratically convergent. State any assumptions that you make.

MATH 640A **Study Guide for the Final Exam** **Fall 2005**

The final exam is Saturday 19 November from 10:10am–12:10, and takes place in our classroom.

Sample question

Dr. S teaches an undergraduate numerical analysis class. This coming quarter she is going to a couple of conferences, and needs someone to teach two class for her. Since you owe her a big favor, you agreed to do it. The topics are: I choose 2 out of

1. Gaussian elimination/ LU decomposition.
2. Neumann series and its alternatives.
3. the Gauss-Seidel iterative method,
4. steepest descent (in the context of solving linear systems).
5. the power method.
6. Schur Factorization.
7. Least-squares problems via QR decompositions.
8. others

She wants you to give her detailed lecture notes so that she knows what you talked about. Please prepare your lecture notes and submit them. Keep in mind:

- Lecture notes are less formal than an essay, but more substantial than an outline. Not everything should be in complete sentences, but key points should be. Mix in formulas and graphs.
- You should have enough detail so that if you get confused about something during the lecture, the answer is in your notes.
- If you are not sure about a formula, write yourself a note to recheck it before class.

MATH 640A **Final Exam** **Fall 2005**

Name: ________________________ Staple this sheet to your exam.
No books, notes, or calculators are allowed.

Question

Dr. S teaches an undergraduate numerical analysis class. This coming quarter she is going to a couple of conferences, and needs someone to teach two class for her. Since you owe her a big favor, you agreed to do it. Choose two out of the following three topics:

1. The Gauss-Seidel iterative method.
2. The power method.
3. The pseudo inverse.

She wants you to give her detailed lecture notes so that she knows what you talked about. Please prepare your lecture notes and submit them. Keep in mind:

- Lecture notes are less formal than an essay, but more substantial than an outline. Not everything should be in complete sentences, but key points should be. Mix in formulas and graphs.
- You should have enough detail so that if you get confused about something during the lecture, the answer is in your notes.
- If you are not sure about a formula, write yourself a note to recheck it before class.
- You can fake the numbers in any examples you give.

IV.F Advising and Supervision of Students

As a director of the Computational Track in the Master's of Science program, I primarily advise master's students, typically 5 each quarter. I have served on 5 doctoral and 1 master's committees while here, all as the College (i.e. outside) representative for engineering students. These committees are listed in Section VIII.

I now run a research group of graduate and undergraduate students, funded by a National Science Foundation CAREER grant (see Section V.F). In the first four quarters of operation, 2 doctoral students, 12 master's students, and 4 undergraduate students have participated, for a total of 30 person-quarters. Although the primary educational goal of the research group is to give the students a research experience, I also try to teach them writing and presentation skills. These skills, which are often neglected in mathematics classes, are nonetheless very important to success in this field. Each week the students write a journal of what they attempted, the methods they used, the outcome, and the next steps. At the weekly group meeting some of the group gives a presentation on their progress on their project. At the end of the quarter they create a more mathematical document as their final report, and give a presentation on it.