Math 2301 **Guide for Test 3**

Here are some sample questions from sections 4.1–4.7. The actual test will be significantly shorter.

1. Find the absolute maximum and minimum values of f on the given interval.

$$f(t) = t\sqrt{4 - t^2} \quad \text{on} \quad [-1, 2]$$

[Tests max and min in 4.1. From Spring 2013 final exam.]

2. Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

$$f(x) = 5 - 12x + 3x^2 \quad \text{on} \quad [1, 3]\,.$$

[Tests Rolle's Theorem in 4.2. From Spring 2013 final exam.]

3. Let f be a continuous function with $f(0) = 3$, $f(2) = 6$, $f'(x) = 0$ for $0 < x < 1$, and $f'(x) < 2$ for $1 < x < 2$. Sketch such a function or explain why it is impossible.
[Tests Mean Value Theorem in 4.2.]

4. Sketch the graph of a single function that has all of the following properties:

   (a) Continuous and differentiable everywhere except at $x = -3$, where it has a vertical asymptote.

   (b) A horizontal asymptote at $y = 1$.

   (c) An x-intercept at $x = -2$.

   (d) A y-intercept at $y = 4$.

   (e) $f'(x) > 0$ on the intervals $(-\infty, -3)$ and $(-3, 2)$.

   (f) $f'(x) < 0$ on the interval $(2, \infty)$.

   (g) $f''(x) > 0$ on the intervals $(-\infty, -3)$ and $(4, \infty)$.

   (h) $f''(x) < 0$ on the interval $(-3, 4)$.

   (i) $f'(2) = 0$.

   (j) An inflection point at $(4, 3)$.

   [Tests curve sketching concepts in 4.1–4.4 without calculations.]

5. (a) Find any vertical asymptotes.
   (b) Find the intervals on which f is increasing or decreasing.
   (c) Find the local maximum and minimum values of f.
   (d) Find the intervals of concavity and the inflection points.
   (e) Use the information from (a)-(d) to sketch the graph.

$$f(x) = xe^{-x}$$

   [Tests curve sketching 4.4 and dependent skills. From Spring 2013 final exam.]

6. Analyze and graph the function $f(x) = x + \dfrac{9}{x}$.
   [Tests curve sketching 4.4 and dependent skills.]

7. What is the maximum vertical distance between the line $y = x + 2$ and the parabola $y = x^2$ for $-1 \leq x \leq 2$?
   [Tests optimization 4.5. From Spring 2013 final exam.]

8. A company wishes to manufacture a box with a volume of 6 m^3 that is open on top and has a square base. The material for the bottom of the box costs \$3 per m^2, while the material for the sides costs \$2 per m^2. Find the dimensions of the box that will lead to minimum total cost. What is the minimum total cost?
   [Tests optimization 4.5.]

9. Use Newton's method with the specified initial approximation x_1 to find x_2, the second approximation to the root of the given equation. Leave the answer as a fraction.

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 = -3\,, \quad x_1 = -3\,.$$

   [Tests Newton's method 4.6. From Spring 2013 final exam.]

10. Find the function f for $x > 0$ that has $f''(x) = x^{-2}$, $f(1) = 0$, and $f(2) = 0$.
    [Test antiderivatives 4.7.]