Math 2301 **Guide for Test 1**

Here are some sample questions from sections 1.3–1.6 and 2.1–2.4. The actual test will be significantly shorter.

1. Consider the function
$$f(x) = \begin{cases} x^2 & \text{if } x \leq -2 \\ Ax & \text{if } x > -2 \end{cases},$$
where A is some constant.
   (a) Find $\lim_{x\to -2^-} f(x)$. Is f continuous from the left at $x = -2$?
   (b) What value of A would make f continuous at $x = -2$?
   (c) Using the value of A that you just found, graph f.

   [Tests one-sided limits (Sections 1.3 and 1.4) and the concept of continuity (1.5).]

2. Use the Intermediate Value Theorem to show that the equation $x^2 = \cos(x)$ has a solution. [Tests the concept of continuity (1.5) and its use in the Intermediate Value Theorem.]

3. Compute the following limits. If you use the squeeze theorem, then indicate the two functions that you are using to squeeze.
   (a) $\lim_{x\to 2} \dfrac{x-2}{x^2-5x+6}$
   (b) $\lim_{x\to 1} \dfrac{\sqrt{x}-1}{x-1}$
   (c) $\lim_{x\to 0} x^2\cos(3/x)$
   (d) $\lim_{h\to 0} \dfrac{x^2-(x-2h)^2}{h}$
   (e) $\lim_{t\to 0} \dfrac{\frac{1}{5+t}-\frac{1}{5}}{t}$
   (f) $\lim_{x\to 2^+} \dfrac{x+2}{x^2-5x+6}$
   (g) $\lim_{x\to -\infty} \dfrac{3x^3-4}{2x^3-2}$
   (h) $\lim_{x\to\infty} \cos(1/x)$
   (i) $\lim_{x\to\infty} (x-x^2)$

   [Tests the computation of limits (1.4, 1.6).]

4. Let $f(x) = -x^2+3$.
   (a) State the definition of the derivative as a limit.
   (b) Using this definition, compute $f'(x)$.
   (c) Find the equation for the tangent line at $x = 2$.

(d) Graph $f(x)$ and the tangent line.

[Tests computation of the derivative from the limit definition (2.2), the use of the derivative to get the slope of the tanget line (2.1), and the construction of a tangent line (2.1).]

5. Find the limit. What derivative does this limit represent?

$$\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

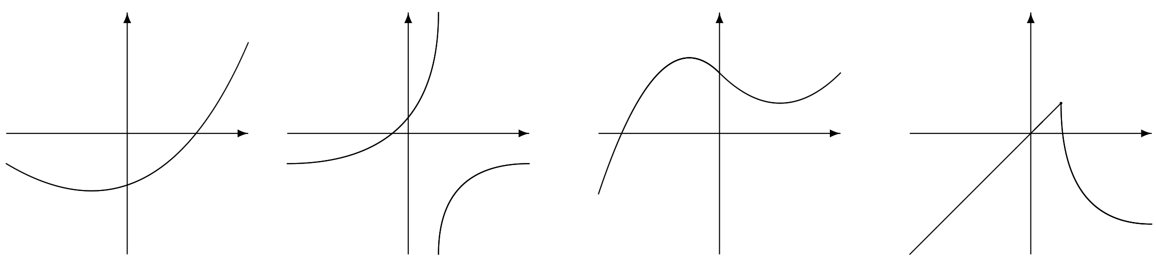
[Tests computation of limits (1.4) and definition of the derivative (2.2). Taken from spring 2013 final exam.]

6. Sketch the graph of a function f for which:
It has a removable discontinuity at $x = 1$, $\lim_{x\to 2^-} f(x) = +\infty$, $\lim_{x\to 2^+} f(x) = -1$, $\lim_{x\to +\infty} f(x) = 3$, $f'(x) > 0$ everywhere it exists.
[Test concept of limits (1.3, 1.6), continuity (1.5), and derivative (2.1, 2.2). Taken from spring 2013 final exam.]

7. Find values for m and b so that $f(x) = \begin{cases} x^2 & \text{if } x \le -2 \\ mx+b & \text{if } x > -2 \end{cases}$ is differentiable at $x = -2$.
[Tests the concept of differentiability (2.2).]

8. Sketch the graph of a function g for which $g(0) = g'(0) = 0$, $g'(-1) = -1$, $g'(1) = 3$, and $g'(2) = 1$.
[Tests graphical understanding of the derivative (2.2).]

9. The graph of a function f is given in each part below. On the same axes, sketch the graph of f'.



[Tests graphical understanding of the derivative (2.2).]

10. Compute the following derivatives:

(a) $f(x) = 2 + x + \dfrac{3}{x} - \sqrt{x} - 5x^7 + x^{3/4} \Rightarrow f'(x) =$

(b) $\dfrac{d}{dx}\left[3\sin(x) + \cot(x) - \sin(7)\right] =$

(c) $D_x\left[(x^9 + x^8 + x^5 + 3)(1 + 2x^2 + 9x^3 - 4x^4)\right] =$

(d) $y = \dfrac{x^3 + x}{x} \Rightarrow \dfrac{dy}{dx} =$

[Tests the basic differentiation formulas (2.3) and product and quotient rules (2.4).]