Math 5600 **Fall 2012**

Homework 9, due Friday 7 December.

1. (50 points) Do this problem as a Good Problem.

   Consider the linear system $A\mathbf{x} = \mathbf{b}$ given by

   $$\begin{bmatrix} 1 & 2 \\ 1.0001 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3.0001 \end{bmatrix}.$$

   (a) Find the condition number of A using $\|\cdot\|_\infty$.

   (b) If we make a small error in A, we may have the system

   $$\begin{bmatrix} 1 & 2 \\ 0.9999 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3.0001 \end{bmatrix}.$$

   Solve this system using five digit rounding, and see what error in $\mathbf{x}$ was caused by the small error in A. Compare this error to the estimated error based on the condition number.

   (c) If we make a small error in $\mathbf{b}$, we may have the system

   $$\begin{bmatrix} 1 & 2 \\ 1.0001 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3.0002 \end{bmatrix}.$$

   Solve this system using five digit rounding, and see what error in $\mathbf{x}$ was caused by the small error in $\mathbf{b}$. Compare this error to the estimated error based on the condition number.

2. (50 points) Consider the matrix A and vector $\mathbf{x}^{(0)}$ given by

   $$A = \begin{bmatrix} 6 & 2 & -1 \\ 2 & 5 & 1 \\ -1 & 1 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{x}^{(0)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

   (a) By hand, do two iterations of the power method starting with $\mathbf{x}^{(0)}$.

   (b) Using MATLAB, do 48 more iterations, and report the results. How close did you get to an actual eigenvalue and eigenvector?

   (c) Use MATLAB to get the LU decomposition of A. By hand, use the LU decomposition to do two iterations of the inverse power method (without shift) starting with $\mathbf{x}^{(0)}$.

   (d) Using MATLAB, do 48 more iterations, and report the results. How close did you get to an actual eigenvalue and eigenvector?

   (e) Using MATLAB to do the computations, select a shift q (by trial and error) and apply the inverse power method starting with $\mathbf{x}^{(0)}$ to find the middle eigenvalue and corresponding eigenvector.