Math 5600 **Fall 2012**

Homework 8, due Friday 2 November.

1. (10 points) Claim your topic for your final project:
   - Edit my user page to add your topic (e.g. 'Secant Method') by your user name. If someone else has already claimed that topic then you need to choose something else.
   - Edit your user page to add a paragraph explaining what you plan to do. Give enough detail that your classmates and I can be sure your project does not overlap with another's.

2. (40 points) Do this problem as a Good Problem, paying attention to the *Graphs* handout.

   Derive the Adams-Bashforth two-step method. Draw a graph to illustrate the derivation.

3. (20 points) To numerically solve an autonomous initial value ODE $y'(t) = f(y(t))$, the following method is proposed:

$$y_{n+1} = y_n + \frac{h}{4}\left[f(y_n) + 3f\left(y_n + \frac{2}{3}hf(y_n)\right)\right].$$

   Determine (and prove) the order of this method.

4. (30 points)

   (a) Complete the following MATLAB code:

```
function [T Y] = backwardeuler(f,tspan,y0,n,tol)
% Solves dy/dt = f(t,y) with initial condition y(a) = y0
% on the interval [a,b] using n steps of the backward Euler method.
% Each step is backward Euler: Y(i+1)=Y(i)+h*f(t(i+1),Y(i+1)).
% To solve for Y(i+1) uses z0=Y(i) and z1=Y(i)+h*f(t(i),Y(i))
% and then runs the secant method on g(z)=z-Y(i)-h*f(t(i+1),z).
% (If f(t(i),Y(i))=0 then it uses z1=z0+h.)
% Inputs: f -- the (scalar) function, as an inline
%         tspan -- a vector [a,b] with the start and end times
%         y0 -- the starting value, y(a)=y0.
%         b -- the right end (ending point) of the interval
%         n -- the number of steps to use; defines the step size.
%         tol -- the secant step runs until |g(z)|<tol.
% Outputs: T -- a n+1 column vector containing the times
%          Y -- a n+1 column vector with the y values
```

   (b) Exercise 30.1 from the MATH 3600 notes (`http://www.math.ohiou.edu/courses/math3600/`) lecture 30 asks you to compare `myeuler` and `mymodeuler` (which are provided). Do that exercise, but also compare with your `backwardeuler`.

   (c) The Wikipedia page `http://en.wikipedia.org/wiki/Stiff_equation` has a motivating example using the equation $y'(t) = -15y(t)$ with $y(0) = 1$. It has a graph demonstrating that the Euler method experiences instability whereas the two-stage Adams-Moulton method does not. Produce a similar graph demonstrating that `myeuler` experiences instability whereas `backwardeuler` does not. Determine whether or not `mymodeuler` experiences instability.