Math 5600 **Fall 2012**
Homework 6, due Friday 12 October.

1. (10 points) Look at the exercise and quiz templates on my user page. Make a simple example and simple quiz on your user page to demonstrate that you can use these templates. Note: the quiz template works on Wikiversity, but not on Wikipedia; you also cannot test the quiz on a preview page.

2. (20 points) Suppose you have the values $f(x_0)$, $f(x_0 + h)$ and $f(x_0 + 2h)$.

   (a) Compute the best estimate for $f'(x_0)$ and an error bound.

   (b) Compute the best estimate for $f''(x_0)$ and an error bound.

3. (40 points) Do this problem as a Good Problem, paying attention to the *Symbols* handout.

   Determine the coefficients a, b, c, and d such that the formula

   $$\int_{-1}^{1} f(x)dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

   is exact for all polynomials of degree 3 or less.

4. (30 points)

   (a) Write a MATLAB function to do the composite trapezoid rule. Start from

   ```
   function x = trapezoid(f,a,b,n)
   % Uses the composite trapezoid rule to approximate int_a^b f(x) dx.
   % Inputs: f -- the function, as an inline
   %         a -- the left end of the interval
   %         b -- the right end of the interval
   %         n -- the number of subintervals to use.
   ```

   Include comments explaining each step.

   (b) Write a MATLAB function to do Romberg's method for integration. Start from

   ```
   function x = romberg(f,a,b,depth)
   % Uses Romberg's method to approximate int_a^b f(x) dx.
   % Inputs: f -- the function, as an inline
   %         a -- the left end of the interval
   %         b -- the right end of the interval
   %         depth -- the number of levels to use.
   % Output: x -- the estimated integral.
   ```

   Have it call your subroutine `trapezoid` to do the trapezoid rule part. Include comments explaining each step.

   (c) Test your routines on $f(x) = x^2 \ln(x)$ on $[1, 1.5]$ for various `n` and `depth`. In particular, compare the performance of the two routines when $n = 2^{depth}$.