Math 4600/5600 **Fall 2012**

Homework 2, due Friday 7 September.

1. (25 points) Write a program to determine the smallest number ϵ so that on your computer $(1+\epsilon)-1 \neq 0$. Include many comments to explain your algorithm. Print and submit the program and the result of running it.

2. (35 points)

   (a) For the quadratic polynomial ax^2+bx+c, the standard quadratic formula says the roots are at
   $$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$
   By rationalizing the numerator we obtain the alternative formula
   $$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\left(\frac{-b \mp \sqrt{b^2-4ac}}{-b \mp \sqrt{b^2-4ac}}\right) = \frac{-2c}{b \pm \sqrt{b^2-4ac}}.$$
   Construct an algorithm that determines each of the two roots using the version of the formula that is most accurate for it.

   (b) Write a MATLAB function program to implement your algorithm. Name/format it as
   `function [x1,x2]=quadroots(a,b,c)`

   (c) Write a MATLAB function program to test your `quadroots` function. Have it call `quadroots` to get x_1 and x_2, and then compute $r_1 = ax_1^2+bx_1+c$ and $r_2 = ax_2^2+bx_2+c$. Name/format it as
   `function [r1,r2]=testquadroots(a,b,c)`
   Use it to check on several quadratic polynomials, including x^2+1, x^2-2x+1, x^2+x-10^{-16}, and x^2-x+10^{-16}.

3. (40 points) Do this problem as a Good Problem, paying attention to the *Logic* handout.

   For the function $f(x)=\cos(x)$ construct the Taylor approximation of degree two $P_2(x)$ around $x_0=0$ and the error term $R_2(x)$. If we use $P_2(\pi/4)$ to approximate $f(\pi/4)$ what bound do we have on the error?