Math 5600 Fall 2012 **Guide for the Final Exam**

The final exam is Friday 14 December 8-10am in our regular classroom. Here are some sample questions.

1. Some questions directly from the three tests.

2. Let
$$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 2 & 1 \\ 1 & 2 & -3 \end{bmatrix}$$

   (a) Use Gaussian elimination without pivoting to factor $A = LU$.

   (b) Use your LU factorization to solve
$$Ax = \begin{bmatrix} 6 \\ 8 \\ 0 \end{bmatrix}.$$

3. Give an example of a matrix that does not have an LU-decomposition but does have a PLU-decomposition.

4. For vectors $(x, y, z) \in \mathbf{R}^3$, determine (and prove) which of the following are norms:

   (a) $|x + y| + |z|$

   (b) $|x| + |y| + |z|$

   (c) $2|x| + 3|y| + 4|z|$

   (d) $2|x| + 3|y|$

5. Suppose an invertible matrix A has an eigenvector $\mathbf{x}$ with eigenvalue λ.

   (a) Show that $\mathbf{x}$ is an eigenvector of $A - qI$ with eigenvalue $\lambda - q$.

   (b) Show that $\mathbf{x}$ is an eigenvector of A^{-1} with eigenvalue λ^{-1}.

   (c) Show that $\mathbf{x}$ is an eigenvector of $(A - qI)^{-1}$ with eigenvalue $(\lambda - q)^{-1}$. (Assume $A - qI$ is invertible.)

6. Your friend did not take any numerical analysis classes, but now needs to do some numerical calculations. They keep having problems that they cannot figure out. For each scenario below

   - explain what is likely causing this problem and
   - explain what to do to fix it.

   (a) They are trying to find the solution to $f(x) = 0$ using Newton's method but the program says "NaN: division by 0 error".

   (b) They construct an interpolating polynomial using the Vandermonde matrix method, but they find it fails to pass through the data points by as much as 0.01, which is too big.

   (c) They want to estimate $f'(x)$ using $(f(x + h) - f(x))/h$. They want it really accurate so they set $h = 10^{-32}$, but then they just get 0 no matter what f and x they try.

   (d) They heard RK4 is the best ODE method to use, but when they try it on their ODE $y' = -50y$, they find the method give $|y_i| \to \infty$.

   (e) They solve $A\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$ but when they check $\|A\mathbf{x} - \mathbf{b}\|_\infty / \|\mathbf{b}\|_\infty$ they get 0.01, which is too big.

   (f) For a fixed large matrix A they need to solve $A\mathbf{x}_i = \mathbf{b}_i$ for many different $\mathbf{b}_i$. They start the program running but it is taking way too long.

   (g) They run the power method to find the largest eigenvalue and its eigenvector. The eigenvalue estimate converges, but the vector produced is not an eigenvector.