Math Math 5600 Fall 2012 **Guide for Test 2**

1. Given the data $\{(x_i, y_i)\} = \{***\}$, construct an interpolating polynomial.

2. Compare the ** and *** methods for constructing an interpolating polynomial with respect to:

   (a) computational cost and

   (b) accuracy.

3. Prove the following theorem:
   Theorem: Let f be a $n+1$ times continuously differentiable function on a closed interval and $P_n(x)$ be a polynomial of degree at most n that interpolates f at $n+1$ distinct points $\{x_i\}_{i=0}^n$ in that interval. Then for each x in the interval there exists ξ in that interval such that
   $$f(x) - P_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}\prod_{i=0}^{n}(x - x_i)\,.$$

4. Suppose you have the values $f(x_0)$, $f(*)$ and $f(*)$.

   (a) Compute the best estimate for $f'(**)$ and an error bound.

   (b) Compute the best estimate for $f''(**)$ and an error bound.

5. Suppose that we have a method $N(h)$ to approximate some quantity M, and we know $M = N(h) + k_1h^2 + k_2h^3 + \cdots$. Given the values $N(*) = **$ and $N(**) = ***$, find the best approximation for M.

6. Suppose we wish to make an approximation $\int_*^{**} f(x)dx \approx c_1 f(**) + c_2 f(x_2)$.

   (a) Determine the best values of c_1, c_2, and x_2.

   (b) Apply your method to $f(x) = ***$ and compute the error.

7. Prove the following theorem:
   Theorem: Suppose $x_1, x_2, \ldots, x_n$ are the roots of the nth Legendre Polynomial $P_n(x)$ and that for each $i = 1, 2, \ldots, n$, the numbers c_i are defined by
   $$c_i = \int_{-1}^{1} \prod_{j=1, j\neq i}^{n} \frac{x - x_j}{x_i - x_j}\, dx\,.$$
   If $P(x)$ is any polynomial of degree less than $2n$, then
   $$\int_{-1}^{1} P(x)dx = \sum_{i=1}^{n} c_i P(x_i)\,.$$