Here are some sample questions. Some topics that we covered are not represented by these questions, but are still fair game.

1. You are reading a biology paper that gives the population model

$$N_{t+1} = \frac{25N_t}{13 + 3N_t^2},$$

where N_t represents the number of rabbits (measured in thousands) at time t (measured in years).

   (a) If this model is accurate, how many rabbits will there be in 10,000,000 years?

   (b) What is the carrying capacity of this environment? Show that if the current population is greater than the carrying capacity then the population will decrease.

   (c) If there are very few rabbits compared to the carrying capacity, then the population should grow exponentially, and the model should behave like $N_{t+1} \approx RN_t$. Determine the effective value of R used in this rabbit model. Explain your reasoning.

2. Consider the difference equation

$$x_{t+1} = \frac{10x_t^2}{9 + x_t^2}.$$

   (a) Use the stability criteria to characterize the stability of its equilibria.

   (b) Use cobwebbing to decide to which value x_t converges as $t \to \infty$ if $x_0 = 0.5$, and if $x_0 = 3$.

3. Solve each differential equation.

   (a) $\dfrac{dy}{dt} = 2\cos(3t)$, where $y(0) = 7$.

   (b) $\dfrac{dy}{dx} = 2y + 1$, where $y_0 = 3$ for $x_0 = 5$.

   (c) $\dfrac{dr}{ds} = \dfrac{\sin(s)}{e^r}$, where $s_0 = 5$ for $r_0 = 7$.

4. Solve the differential equation $\dfrac{dN}{dt} = 5(N-2)(N-3)$, where $N(0) = 7$.

5. Suppose we know that $\dfrac{dN}{dt} = N(N-1)(N-3)$. (Do **not** try to solve this equation.)

   (a) Find the equilibria, and classify each equilibrium as stable or unstable.

   (b) If $N(0) = 2$ then what value will $\lim\limits_{t\to\infty} N(t)$ have? Justify your answer.

6. That one pond in your home town has become contaminated with a (soluble) polutant at concentration 3 grams/liter. The pond has volume of 50000 liters. Clean rainwater flows into the pond and the average flow rate of the stream leading out of its spillway is 4 liter/minute. How long will it take until the concentration lowers to 1 grams/liter?