Math 444/544 **Guide for Test 2**

Test 2 is on **Monday 1 November**; note the change in day. I attempted to outline my test and give you an outline, but there is no warranty.

1. Suppose we wish to make an approximation $\int_*^{**} f(x)dx \approx c_1 f(**) + c_2 f(x_2)$.
   (a) Determine the best values of c_1, c_2, and x_2.
   (b) Apply your method to $f(x) = ***$ and compute the error.

2. Suppose that we have a method $N(h)$ to approximate some quantity M, and we know $M = N(h) + k_1 h^2 + k_2 h^3 + \cdots$. Given the values $N(*) = **$ and $N(**) = ***$, find the best approximation for M.

3. To numerically solve an initial value ODE $y'(t) = f(t, y(t))$ with $y(t_0) = y_0$, the following method is proposed:
$$y_{n+1} = \quad \text{**RK type**}\,.$$
   (a) Determine (and prove) the order of this method.
   (b) Determine the stability domain of this method.
   (c) Given $f(t, y) = ***$, $y(t_0) = y_0 = **$, and $h = ***$, compute y_2.

4. To numerically solve an initial value ODE $y'(t) = f(t, y(t))$ with $y(t_0) = y_0$, the following method is proposed:
$$y_{n+1} = \quad \text{**multistep type**}\,.$$
   (a) Determine (and prove) the order of this method.
   (b) Determine if the method is stable.
   (c) Given $f(t, y) = ***$, $y(t_0) = y_0 = **$, $y(t_0 + h) = y_1 = ***$, and $h = ***$, compute y_3.

5. **Math 444 students:** Make sure you wrote your name on the test.

   Math 544 students: Prove the following theorem:

   Theorem: Suppose $x_1, x_2, \ldots, x_n$ are the roots of the nth Legendre Polynomial $P_n(x)$ and that for each $i = 1, 2, \ldots, n$, the numbers c_i are defined by
$$c_i = \int_{-1}^{1} \prod_{j=1, j\neq i}^{n} \frac{x - x_j}{x_i - x_j} dx.$$
   If $P(x)$ is any polynomial of degree less than $2n$, then
$$\int_{-1}^{1} P(x)dx = \sum_{i=1}^{n} c_i P(x_i)\,.$$