Math 444/544 Fall 2010 **Guide for Test 1**

Here are some sample questions. I attempted to outline my test and give you an outline, but there is no warranty.

1. We wish to construct a quadratic polynomial approximation for the function $f(x) = **$ on the interval $[*, *]$.

   (a) Construct the Taylor approximation of degree two $P_2(x)$ around $x_0 = ***$ and the error term $R_2(x)$. If we use $P_2(****)$ to approximate $f(****)$ what bound do we have on the error?

   (b) Construct the Lagrange interpolating polynomial of degree two $Q_2(x)$ using the points *****, and the error term. If we use $Q_2(****)$ to approximate $f(****)$ what bound do we have on the error?

   (c) Which method is better? Why?

2. Consider the fixed-point iteration

$$x_{n+1} = ****\tag{1}$$

   (a) Apply the iteration twice, starting at $x_0 = **$.

   (b) Determine the fixed point(s) of this iteration.

   (c) Determine which initial x_0 will converge to which fixed point, and which will diverge.

   (d) For one of the fixed points, determine the order of convergence.

3. To compute the solution of $*a**b***$ you could use the formmula

$$***\tag{2}$$

   or the formula

$$***\tag{3}$$

   Write an algorithm that, for given input a and b, chooses the formula that minimizes loss of significance. Justify your choice.

4. (a) State the conditions under which Newton's method for solving $f(x) = 0$ will have quadratic convergence.

   (b) **Math 444 students:** Make sure you wrote your name on the test.

   Math 544 students: Prove that Newton's method does indeed have quadratic convergence under these conditions.