Math 410/510 **Spring 2010** **Guide for Test 3**

- The test is in class on Friday 28 May and covers Chapters 7 and 8. You must notify me in advance if you have to miss the test.
- Only Math 510 students are responsible for the material in Sections 7.10 and 8.6. They will have an extra question to prove a lemma or theorem in those sections or a theorem from another section.

Here are some sample test questions/topics. Things written in [brackets] are comments.

1. (a) State the Cayley-Hamilton Theorem.
   (b) Demonstrate that this theorem is true for the matrix $\mathbf{A} = ***$.
   (c) Compute $\mathbf{f}(\mathbf{A}) = ***$ for the matrix $\mathbf{A} = ***$. [Could have distinct, repeated, or complex eigenvalues.]
2. [properties of the exponential, sections 7.8 and 7.9] Show that $e^{-\mathbf{A}t} = \left(e^{\mathbf{A}t}\right)^{-1}$.
3. Put the following system into fundamental form: *** [section 8.3]
4. Solve the system $\dot{\mathbf{x}}(t) = \begin{bmatrix} * & * \\ * & * \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} * \\ * \end{bmatrix}$ with initial conditions $\mathbf{x}(*) = \begin{bmatrix} * \\ * \end{bmatrix}$ for $\mathbf{x}(t)$. [2×2 constant coefficient, like section 8.4. These are long, hard problems and there will definitely be one on the test.]
5. Consider the linear differential system $\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{f}(t)$ with initial conditions $\mathbf{x}(t_0) = \mathbf{c}$.
   (a) Define the transition matrix $\mathbf{\Phi}(t, t_0)$ for this system.
   (b) State the solution to this system in terms of the transition matrix.
   (c) If $\mathbf{A} = ***$, determine if $\mathbf{\Phi}(t, t_0) = ***$ is the transition matrix.