Math 410/510 **Spring 2010** **Guide for Test 2**

- The test is in class on Friday 7 May and covers Chapters 3, 5, and 6. You must notify me in advance if you have to miss the test.
- Only Math 510 students are responsible for the material in Sections 3.6 and 5.7. They will have an extra question to prove a lemma or theorem in those sections or a theorem from another section.

Here are some sample test questions/topics. Things written in [brackets] are comments.

1. [Probably a 2×2 with some of these questions and a 3×3 with others of these questions.] Let $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ -4 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

   (a) Compute the determinant of $\mathbf{A}$ using [cofactor expansion or elementary row operations and pivotal condensation].

   (b) Compute $\mathbf{A}^{-1}$ using elementary row operations (Gaussian Elimination).

   (c) Compute $\mathbf{A}^{-1}$ using the adjugate.

   (d) Check that the matrices you computed above satisfy the definition of the inverse of $\mathbf{A}$.

   (e) Use the inverse you computed above to solve $\mathbf{Ax} = \mathbf{b}$.

   (f) Compute the $\mathbf{LU}$ decomposition of $\mathbf{A}$ and use it to solve $\mathbf{Ax} = \mathbf{b}$.

   (g) Use Cramer's rule to solve $\mathbf{Ax} = \mathbf{b}$.

   (h) Find all the eigenvalues of $\mathbf{A}$, with multiplicities.

   (i) Find the eigenvectors of $\mathbf{A}$. [There may be fewer linearly independent eigenvectors than the size (order) of $\mathbf{A}$.]

   (j) Show that the eigenvectors that you found above satisfy the definition of an eigenvector.

2. [More theoretical questions.]

   (a) [properties of the inverse from section 3.4] Show that $(\mathbf{BC})^{-1} = \mathbf{C}^{-1}\mathbf{B}^{-1}$.

   (b) [properties of determinant from section 5.3] Show that the determinant of an upper triangular matrix is the product of the elements on the main diagonal.

   (c) [properties of eigenvalues and vectors from 6.4] Show that if λ is an eigenvalue of $\mathbf{B}$ and $\mathbf{B}$ is invertible, then $1/\lambda$ is an eigenvalue of $\mathbf{B}^{-1}$.