

Here are some sample questions from Sections 4-6, 5-1, 5-2, and 5-4. Some topics that we covered are not represented by these questions, but are still fair game.

1. A weather balloon is rising vertically at the rate of 5 meters per second. An observer is standing on the ground 300 meters from the point where the balloon was released. At what rate is the distance between the observer and the balloon changing when the balloon is 400 meters high?
2. Let  $f(x) = \frac{x^3}{3} - 2x^2 + 3x + 1$ .
  - (a) Find the domain of  $f$ , any discontinuities, any asymptotes, and its  $y$ -intercept.
  - (b) Find the critical values, extrema, and intervals of increase or decrease.
  - (c) Find the intervals where it is concave up, and any inflection points.
  - (d) Graph  $f(x)$ , labeling the points that you found above.
3. Sketch the graph of a single function that has all of the following properties:
  - (a) Continuous and differentiable everywhere except at  $x = -3$ , where it has a vertical asymptote.
  - (b) A horizontal asymptote at  $y = 1$ .
  - (c) An  $x$ -intercept at  $x = -2$ .
  - (d) A  $y$ -intercept at  $y = 4$ .
  - (e)  $f'(x) > 0$  on the intervals  $(-\infty, -3)$  and  $(-3, 2)$ .
  - (f)  $f'(x) < 0$  on the interval  $(2, \infty)$ .
  - (g)  $f''(x) > 0$  on the intervals  $(-\infty, -3)$  and  $(4, \infty)$ .
  - (h)  $f''(x) < 0$  on the interval  $(-3, 4)$ .
  - (i)  $f'(2) = 0$ .
  - (j) An inflection point at  $(4, 3)$ .
4. Analyze and graph the function  $f(x) = x + \frac{9}{x}$ .