Math 446/546 **Spring 2009**

Homework 7, due Friday 29 May.

Consider the nonlinear system of equations

$$\begin{cases} 3x_1 - \cos(x_2 x_3) - 1/2 = 0 \\ 4x_1^2 - 625x_2^2 + 2x_2 - 1 = 0 \\ e^{-x_1 x_2} + 20x_3 + (10\pi - 3)/3 = 0 \end{cases}$$

with initial guess

$$\mathbf{x}^{(0)} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

1. (a) (10 points) Solve the first equation for x_1, the second for x_2 (leaving the $625x_2^2$ part on the other side), and the third for x_3, to obtain a functional iteration $\mathbf{x}^{(k)} = \mathbf{G}(\mathbf{x}^{(k-1)})$.

   (b) (10 points) Apply this iteration twice, starting at $\mathbf{x}^{(0)}$.

   (c) (10 points) Try to determine if it will converge using Theorem 10.6. If you are unable to show convergence, explain the difficulties you encountered.

2. (a) (20 points) Apply two steps of Newton's method starting at $\mathbf{x}^{(0)}$ to obtain $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$.

   (b) (20 points) Starting with $\mathbf{x}^{(0)}$ and $\mathbf{x}^{(1)}$, apply one step of Broyden's method to obtain a different $\tilde{\mathbf{x}}^{(2)}$.

3. (30 points) Do this problem as a Good Problem.
   Apply two steps of the steepest descent technique starting at $\mathbf{x}^{(0)}$.