Math 446/546 **Spring 2009**
Homework 5, due Friday 8 May.

1. (30 points) Do this problem as a Good Problem, paying attention to the *Graphs* handout.

   State the Gerschgorin Circle theorem and then use it to estimate the eigenvalues of

   $$A = \begin{bmatrix} 10 & -1 & 0 & 1 \\ 0.2 & 8 & 0.2 & 0.2 \\ 1 & 1 & 2 & 1 \\ -1 & -1 & -1 & -11 \end{bmatrix}.$$

   Sketch the region where the eigenvalues are contained.

2. (50 points) Consider the matrix A and vector $\mathbf{x}^{(0)}$ given by

   $$A = \begin{bmatrix} 6 & 2 & -1 \\ 2 & 5 & 1 \\ -1 & 1 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{x}^{(0)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

   (a) By hand, do two iterations of the power method starting with $\mathbf{x}^{(0)}$.

   (b) Using MATLAB, do 48 more iterations, and report the results. How close did you get to an actual eigenvalue and eigenvector?

   (c) Use MATLAB to get the LU decomposition of A. By hand, use the LU decomposition to do two iterations of the inverse power method (without shift) starting with $\mathbf{x}^{(0)}$.

   (d) Using MATLAB, do 48 more iterations, and report the results. How close did you get to an actual eigenvalue and eigenvector?

   (e) Using MATLAB to do the computations, select a shift q (by trial and error) and apply the inverse power method starting with $\mathbf{x}^{(0)}$ to find the middle eigenvalue and corresponding eigenvector.

3. (20 points) Edit the Wikipedia page that you have been working on to incorporate the changes you proposed (after adjusting for feedback you received). Turn in a description of the weakness you had identified, your proposed improvement, feedback you received through the discussion page, and the changes you ended up doing. Print the portion of the page you edited and highlight your changes.