Math 446/546 **Spring 2009**
Homework 4, due Tuesday 28 April.

1. (40 points) Consider the system

$$\begin{bmatrix} 4 & 1 & -1 \\ -1 & 3 & 1 \\ 2 & 2 & 5 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 5 \\ -4 \\ 1 \end{bmatrix}$$

and the starting guess $\mathbf{x}^{(0)} = \mathbf{0}$.

   (a) Do two iterations of the Jacobi iteration. Apply Theorem 7.19 to determine if this iteration will converge.

   (b) Do two iterations of the Gauss-Seidel iteration. Apply Corollary 7.20 to obtain a bound on $\|\mathbf{x} - \mathbf{x}^{(2)}\|$.

2. (30 points) Consider the linear system $A\mathbf{x} = \mathbf{b}$ given by

$$\begin{bmatrix} 1 & 2 \\ 1.0001 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3.0001 \end{bmatrix}.$$

   (a) Find the condition number of A using $\|\cdot\|_\infty$.

   (b) If we make a small error in A, we may have the system

$$\begin{bmatrix} 1 & 2 \\ 0.9999 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3.0001 \end{bmatrix}.$$

   Solve this system using five digit rounding, and see what error in $\mathbf{x}$ was caused by the small error in A.

   (c) If we make a small error in $\mathbf{b}$, we may have the system

$$\begin{bmatrix} 1 & 2 \\ 1.0001 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3.0002 \end{bmatrix}.$$

   Solve this system using five digit rounding, and see what error in $\mathbf{x}$ was caused by the small error in $\mathbf{b}$.

3. (20 points) Do this problem as a Good Problem, paying attention to the *Symbols* handout.

   Consider the linear system $A\mathbf{x} = \mathbf{b}$ given by

$$\begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

   Perform two steps of the Conjugate gradient method starting with $x_1 = 2$ and $x_2 = 1$ to get an approximate solution.

4. (10 points) Adjust your proposed changes to a Wikipedia page, based on my comments on your homework 3. Edit the **discussion** page associated to the Wikipedia page with a description of your proposed changes and your rational. Print the portion of the discussion page with your proposal.