Math 446/546 **Spring 2009**

Homework 3, due Tuesday 21 April.

1. (30 points) Do this problem as a Good Problem, paying attention to the *Logic* handout.

   Define $\|\cdot\|_1$ on $\mathbb{R}^n$ by $\|\mathbf{x}\|_1 = \sum_{i=1}^n |x_i|$.

   (a) Compute $\|(1, 2, 3.4, 7.1)^t\|_1$.

   (b) Verify that $\|\cdot\|_1$ is a norm on $\mathbb{R}^n$.

   (c) Prove that $\|\mathbf{x}\|_2 \leq \|\mathbf{x}\|_1$ for all $\mathbf{x} \in \mathbb{R}^n$

2. (30 points) Define the "Frobenius norm" $\|\cdot\|_F$ on $n \times n$ matrices by

$$\|A\|_F = \left(\sum_{i=1}^n \sum_{j=1}^n |a_{ij}|^2 \right)^{1/2}.$$

   (a) Compute

$$\left\| \begin{bmatrix} 4 & -1 & 7 \\ -1 & 4 & 0 \\ -7 & 0 & 4 \end{bmatrix} \right\|_F.$$

   (b) Verify that $\|\cdot\|_F$ is a matrix norm.

   (c) Show that $\|A\|_2 \leq \|A\|_F \leq n^{1/2}\|A\|_2$ for all matrices A.

3. (30 points) Let

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & -2 & 3 \\ 2 & 0 & 4 \end{bmatrix}.$$

   (a) Compute the eigenvalues and eigenvectors of A.

   (b) Find the spectral radius of A.

   (c) Determine if A is convergent.

   (d) Compute $||A||_2$. You can use the MATLAB command `eig` to compute the eigenvalues for this part.

4. (10 points) Explore the Wikipedia pages on the topics that we cover in chapter 6 and 7. Find something that is incorrect, incomplete, or poorly explained. Write down what you found, where you found it, and how you propose to change it to make it better.