Math 446/546 **Spring 2009** **Guide for Test 3**

Test 3 is on Monday May 18, and covers Chapter 9. Bring a calculator for arithmetic. Here are some sample questions from Chapter 9.

1. Consider the matrix A with LU decomposition

$$A = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ * & 1 & 0 \\ * & * & 1 \end{bmatrix} \begin{bmatrix} * & * & * \\ 0 & * & * \\ 0 & 0 & * \end{bmatrix} = LU$$

and the vectors

$$\mathbf{x}_1 = \begin{bmatrix} * \\ * \\ * \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} * \\ * \\ * \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} * \\ * \\ * \end{bmatrix}, \quad \text{and} \quad \mathbf{x}^{(0)} = \begin{bmatrix} * \\ * \\ * \end{bmatrix}.$$

(a) Use the Gerschgorin Circle theorem to estimate the eigenvalues of A.

(b) Verify that $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ are eigenvectors of A and find their corresponding eigenvalues.

(c) Do two iterations of the power method starting with $\mathbf{x}^{(0)}$. If you kept going, what would you eventually get?

(d) Using the LU decomposition, do two iterations of the inverse power method (without shift) starting with $\mathbf{x}^{(0)}$. If you kept going, what would you eventually get?

(e) Give an algorithm based on the power method or inverse power method for computing the middle eigenvalue and its eigenvector. Prove that your algorithm converges, stating any assumptions that you need.

(f) Use Householder's method to bring A to tridiagonal form.

(g) Starting with the tridiagonal form, do one step of the (unshifted) QR iteration for finding eigenvalues. Describe how and why shifting is used within this algorithm. Describe how the QR iteration would continue. If you kept going, what would you eventually get?

2. **Math 446 students:** Make sure you wrote your name on the test.

Math 546 students: The book has the following theorem:

Theorem: [**either Theorem 9.13 (first part) or 9.14 **]

Prove this theorem. If you use any other theorems from the book during your proof, then you need to state those theorems.