

Math 444/544**Guide for the Final Exam**

The final exam is Tuesday 18 November 8-10am in our regular classroom.

The exam is cumulative, but the questions from Chapters 1–3 will come directly from the tests you already took on those chapters. In Chapter 4 we cover Sections 4.1–4.5 and 4.6. Here are some sample questions from Chapter 4.

1. For the function $f(x) = **$, compute the centered difference approximation to $f'(**)$ using $h = **$. State the error term for the centered approximation, and then use it to get a bound on your error.
2. Consider the integral $\int_*^{**} f(x)dx$.
 - (a) Use the trapezoid rule to approximate it. State the general error term for the trapezoid rule and then use it to get a bound on your error.
 - (b) Use Simpson's rule to approximate it, and state the general error term for this rule.
 - (c) Use the composite trapezoid rule with *** intervals to approximate it. State the general error term for this rule and then use it to get a bound on your error.
 - (d) Use the composite Simpson's rule with *** intervals to approximate it, and state the general error term for this rule.
3. Suppose we wish to make an approximation $\int_0^1 f(x)dx \approx c_1f(0) + c_2f(x_2)$. Determine the best values of c_1 , c_2 , and x_2 .
4. Suppose that we have a method $N(h)$ to approximate some quantity M , and we know $M = N(h) + k_1h^2 + k_2h^3 + \dots$. Given the values $N(*) = **$ and $N(**) = ***$, find the best approximation for M .
5. **Math 444 students:** Make sure you wrote your name on the test.

Math 544 students: The book has the following theorem:

Theorem: Suppose x_1, x_2, \dots, x_n are the roots of the n th Legendre Polynomial $P_n(x)$ and that for each $i = 1, 2, \dots, n$, the numbers c_i are defined by

$$c_i = \int_{-1}^1 \prod_{j=1, j \neq i}^n \frac{x - x_j}{x_i - x_j} dx.$$

If $P(x)$ is any polynomial of degree less than $2n$, then

$$\int_{-1}^1 P(x)dx = \sum_{i=1}^n c_i P(x_i).$$

Prove this theorem. If you use any other theorems from the book during your proof, then you need to state those theorems.