MATH 640A **Study Guide for Midterm Exam** **Fall 2005**

When: ____________________________

Where: ______________ Morton.

What is covered: Chapters 1, 2, and 3.

General principles:

- Know the core concepts thoroughly. This is not a test of your ability to memorize peripheral details.
- Calculators are forbidden and no formulas will be provided. However, the numbers will be rigged so that you do not need a calculator, and formulas can be purchased during the test.
- You should be able to **do** any of the methods we have learned. Much of the test will be straightforward computation.
- You should be able to **state** the definitions and theorems that we have encountered. See the section notes for which ones you need to be able to prove.
- You should be able to **determine** which method is appropriate for a given situation. That means you should be able to **explain** the advantages and disadvantages of each method.

Section notes:

These notes are to help guide your study, by listing some topics you definitely need to know and some that you can ignore. They are not a list of all that you need to know.

Section 1.1 Know the theorems from Calculus and Taylor's theorem with Lagrange remainder. You can ignore Taylor's theorem with integral remainder and Taylor's theorem in two variables.

Section 1.2 Ignore the mean-value theorem for integrals and the implicit function theorem.

Section 1.3 Know the statements of the theorems and the proof for the theorem on stable difference equations, but not the proof for the theorem on null space.

Section 2.1 Know the terminology and be able to do floating-point error analysis under the assumption of no cancellation. Know the statement of the theorem on relative roundoff error in adding but not the proof.

Section 2.2 Be able to detect the loss of significance and work around it. Ignore interval arithmetic.

Section 2.3 Ignore condition number of a matrix.

Section 3.1 Know bisection thoroughly.

Section 3.2 Know Newton's method thoroughly, including when it fails, the proof for quadratic convergence, and how to do it on systems. Ignore implicit functions.

Section 3.3 Know the secant method, including error analysis.

Section 3.4 Know how to use the contractive mapping theorem but not its proof. Be able to test for contraction using the derivative. Be able to compute order of convergence.

Section 3.5 Know the theorems on algebra and localization of roots, but not their proofs. Know Horner's algorithm and how to use it with Newton's method and deflation to find all the roots. Know the theorem on successive Newton iterates but not the proof. Ignore Bairstow's method and Laguerre's method.

Section 3.6 Be able to set up the homotopy for a system with one or two variables, and describe how to use it.

MATH 640A **Midterm Exam, Thursday 16 October** **Fall $\boxed{2003}$**

Name: ______________________________ Staple this sheet to your exam.

No books, notes, or calculators are allowed.

Show your work and **give reasons** for your conclusions.

1. (10 points) For small values of x, the approximation $\cos(x) \approx 1 - x^2/2$ can be used. Bound the error from using this approximation at $x = 1/2$.

2. (10 points) Construct an **unstable** recurrence of the form

$$x_{i+1} = Ax_i + Bx_{i-1}.$$

   Determine all sequences $\{x_i\}_{i=0}^{\infty}$ that satisfy your recurrence.

3. (10 points) Let a, b, c, and d be positive machine numbers on a machine with unit roundoff ϵ. Estimate the relative error in computing $(a + b) + (c + d)$ and the relative error in computing $((a + b) + c) + d$. Is there any reason to prefer one of these methods for computing the sum?

4. (10 points) Discuss the calculation of e^{-x} for $x > 0$ from the series

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots .$$

   Suggest a better way, assuming that the system function e^x is **not** available.

5. (30 points) Show that Newton's method is quadratically convergent. State any assumptions that you make.

6. (30 points) An undergraduate has just joined your research group. She is eager and smart, but does not know anything about finding roots. You have software to do Bisection, Newton's method, and the Secant method, but you need to teach her when to use each method.

   (a) Explain when to use Bisection, and give an example.

   (b) Explain when to use Newton's method, and give an example.

   (c) Explain when to use the Secant method, and give an example.