

The second test is in class on Friday 7 October.

Here are some sample questions, so that you have an idea of what to expect.

1. (a) Let $a_n = \frac{n^2 + 3}{4n^2}$
 - i. Find a so that $\lim_{n \rightarrow \infty} a_n = a$.
 - ii. Find N so that $|a_n - a| < \epsilon$ whenever $n > N$.
 - iii. Compute $\lim_{n \rightarrow \infty} n(a_n - 1/4)$
- (b) The cells in that slime near Fargo reproduce (by splitting) every 2.642 hours. FEMA found a laser that will kill them, and is killing 1,000,000 of the cells per hour.
 - i. Write a recurrence representing the number of cells.
 - ii. If there are now 3,000,000 cells, is Fargo doomed or saved?
2. You are reading a biology paper that gives the population model

$$N_{t+1} = \frac{25N_t}{13 + 3N_t^2},$$

where N_t represents the number of rabbits (measured in thousands) at time t (measured in years).

- (a) If this model is accurate, how many rabbits will there be in 10,000,000 years?
- (b) What is the carrying capacity of this environment? Show that if the current population is greater than the carrying capacity then the population will decrease.
- (c) If there are very few rabbits compared to the carrying capacity, then the population should grow exponentially, and the model should behave like $N_{t+1} \approx RN_t$. Determine the effective value of R used in this rabbit model. Explain your reasoning.
3. Compute the following limits. If you use the sandwich theorem, then indicate the two functions that you are using to sandwich.
 - (a) $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 5x + 6}$
 - (b) $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1}$
 - (c) $\lim_{x \rightarrow -\infty} \frac{3e^{3x} - 4}{2e^{3x} - 2}$
 - (d) $\lim_{x \rightarrow 0} x^2 \cos(3/x)$
4. Consider the function

$$f(x) = \begin{cases} x^2 & \text{if } x \leq -2 \\ Ax & \text{if } x > -2 \end{cases},$$

where A is some constant.

- (a) Find $\lim_{x \rightarrow -2^-} f(x)$. Is f continuous from the left at $x = -2$?
- (b) What value of A would make f continuous at $x = -2$?
- (c) Using the value of A that you just found, graph f .
5. Use the Intermediate Value Theorem to show that the equation $x^2 = \cos(x)$ has a solution.