Fargo Follow up:

You may have tried to solve the problem in one of three different ways:

Strategy 1. You may have tried to figure out how many cells would be in a circle of radius 50m, how many cells would be in a circle of radius 100 km, and then how much the population increases in one hour. Given a rule for computing the population after it grows for one hour, there are a couple of methods for determining when the slime reaches Fargo. One way would be to compute the population on an hour by hour basis, and note when the population exceeded the number needed to fill a radius of 100 km. Another way would be to find a rule that gave the population after n hours of growth and, by using this rule, find the smallest value of n for which the population exceeded the number needed to fill a circle of radius 100 km.

Let us work through the last option in detail. From the information given, we know that a cell has radius .001 meters and hence has area $(.001)^2\,\pi$ square meters. It follows that the number of cells in a circle of radius 50 meters is

$$\frac{50^2\pi \text{ sq- m}}{(.001^2)\,\pi \text{ sq-m/cell}} = 2.5 \times 10^9 \text{ cells}$$

and that the number of cells in a circle of radius 100 km is

$$\frac{(100000)^2\,\pi \text{ sq- m}}{(.001^2)\,\pi \text{ sq-m/cell}} = 1.0 \times 10^{16} \text{ cells}$$

Now let's go to the results of the population experiments and divide the new population by the old.

initial population	population after one hour	ratio
1000	1301	1.301
2000	2605	1.302
3000	3897	1.299
4000	5201	1.300

The results in the table suggest that

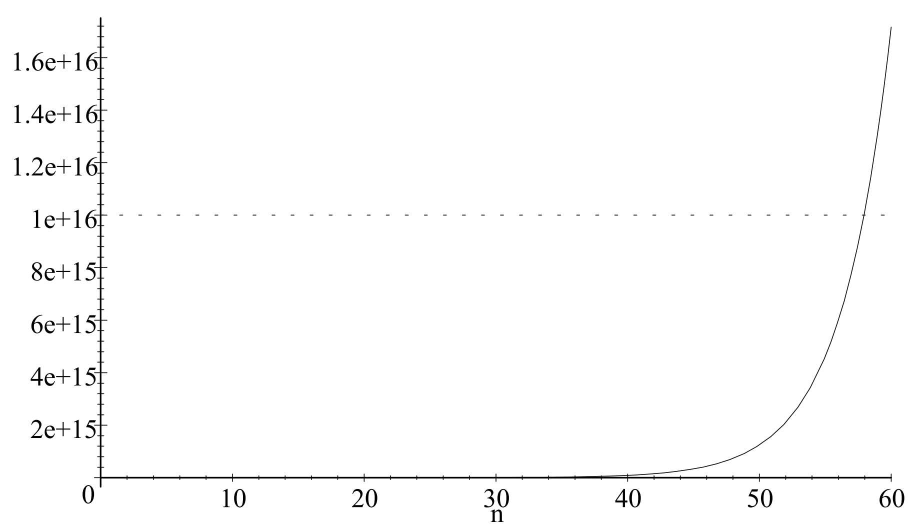
$$(\text{population after one hour}) = (1.3)\,(\text{original population})$$

(Check this by looking at the ratio of the population after one hour to the population after two hours.)

If we call the time the 50 meter patch is first observed time 0 and let $p(n)$ denote the size of the population after n hours, we can compute the values for $p(1), p(2), \ldots, p(n)$, as follows

$$
\begin{aligned}
p(1) &= (1.3)\,p(0) = (1.3)\left(2.5\times 10^{9}\right) \\
p(2) &= (1.3)\,p(1) = (1.3)\left[(1.3)\left(2.5\times 10^{9}\right)\right] = (1.3)^{2}\left(2.5\times 10^{9}\right) \\
p(2) &= (1.3)\,p(2) = (1.3)\left[(1.3)^{2}\left(2.5\times 10^{9}\right)\right] = (1.3)^{3}\left(2.5\times 10^{9}\right) \\
&\vdots \\
p(n) &= (1.3)\,p(n-1) = (1.3)\left[(1.3)^{n-1}\left(2.5\times 10^{9}\right)\right] = (1.3)^{n}\left(2.5\times 10^{9}\right)
\end{aligned}
$$

So 'all we have to do' is find the first n such that $(1.3)^n\left(2.5\times 10^9\right) > 1.0\times 10^{16}$. If we knew how to compute logarithms, we could solve this directly. Otherwise, we have to guess. We can estimate n by looking at the graph of $(1.3)^n\left(2.5\times 10^9\right)$. The graph is sketched below; it suggests that $n = 58$ might be a good guess.

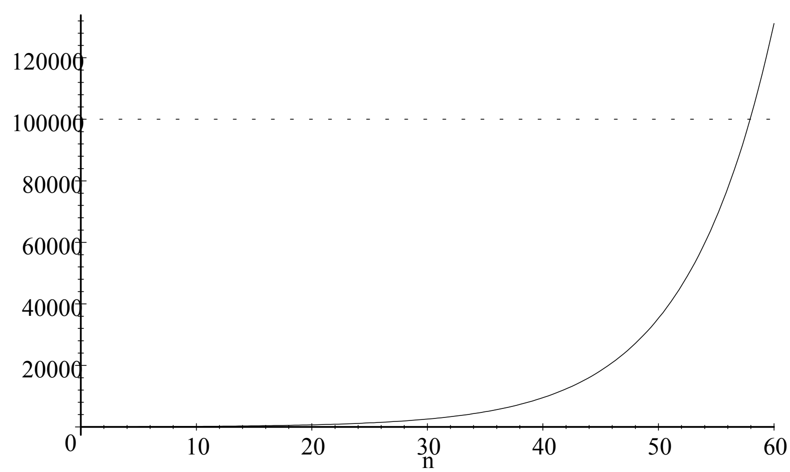


Note that $(1.3)^{57}\left(2.5\times 10^9\right) = 7.8111\times 10^{15}$ and $(1.3)^{58}\left(2.5\times 10^9\right) = 1.0154\times 10^{16}$, so the mold flattens Fargo 58 hours after it was discovered.

Strategy 2: This approach is similar to the preceding approach. This time we look at how the radius is changing as a function of time. Observe that if we look at the ratio of the new radius to the old radius, we get a value close to 1.140. Now, if we let $r(n)$ stand for the radius after n hours, we get that $r(0) = 50$ and we are looking for the first value of n for which $r(n) \geq 100000$. (Note that we have converted kilometers to meters.) Observe that

$$\begin{aligned}
r(1) &= (1.140)\, r(0) = (1.140)(50) \\
r(2) &= (1.140)\, r(1) = (1.140)\,[(1.140)(50)] = (1.140)^2 (50) \\
r(3) &= (1.140)\, r(2) = (1.140) \left[(1.140)^2 (50)\right] = (1.140)^3 (50) \\
&\vdots \\
r(n) &= (1.140)\, r(n-1) = (1.140) \left[(1.140)^{n-1} (50)\right] = (1.140)^n (50)
\end{aligned}$$

As before, 'all we have to do' is find the first value of n for which $r(n) = (1.140)^n (50) \geq 100000$. If we don't have logarithms available to us, we can once again look at the graph of $r(n.)$

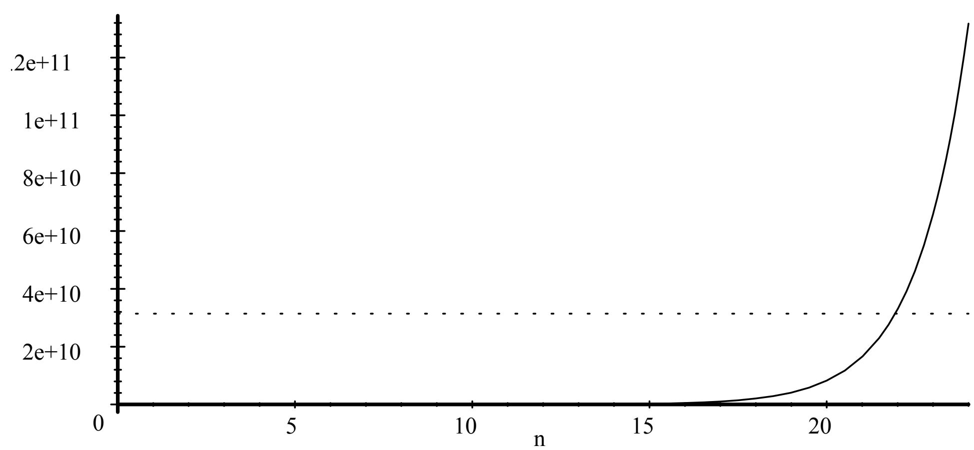


As before, $n = 58$ looks like a reasonable value. Since $(1.140)^{58} (50) = 99874$, it looks like Fargo is flattened in slightly less than 58 hours. (How does 1.140 relate to 1.3 ?)

Strategy 3: Yet another strategy could be based on rate at which the cells reproduce. One of the experiments indicated that the cells reproduced at the rate of once every 2.642 hours. This would mean that the area of the patch would double every 2.642 hours. One could solve the problem by figuring out how many times the patch would have to double in size before reaching Fargo. If we let $s(n)$ denote the area of the patch after it has doubles in size n-times, then we have that

$$\begin{aligned}
s(1) &= (2)\, s(0) = (2)\left(50^2\pi\right) \\
s(2) &= (2)\, s(1) = (2)\left[(2)\left(50^2\pi\right)\right] = (2)^2\left(50^2\pi\right) \\
s(3) &= (2)\, s(2) = (2)\left[\left(2^2\right)\left(50^2\pi\right)\right] = (2)^3\left(50^2\pi\right) \\
&\vdots \\
s(n) &= (2)\, s(n-1) = (2)\left[\left(2^{n-1}\right)\left(50^2\pi\right)\right] = 2^n\left(50^2\pi\right)
\end{aligned}$$

Once again, 'all we have to do' is find the least n for which $s(n) = 2^n\left(50^2\pi\right) > (100000)^2\,\pi = 3.1416 \times 10^{10}$ and then multiply that n by the length of time it takes for the patch to double in size. To estimate n, we look at the graph of $s(n)$.



It looks like $n = 22$ might be a good guess. Since $2^{21}\left(50^2\pi\right) = 1.6471 \times 10^{10}$ and $2^{22}\left(50^2\pi\right) = 3.2942 \times 10^{10}$, 22 is the sought after value of n. Now $22 \times 2.642 = 58.124$, so once again, it looks like Fargo is slime fodder in approximately 58 hours.

Note: By using logarithms, which will be discussed in a later section, one can get exact values for n in each of the above approaches. The slime gets to Fargo in 57.94 hours.