Implicit Differentiation [1]

In this exercise, we will graphically explore the idea behind implicit differentiation. If this MATLAB exercise is being counted in your grade, then be sure to print the figures for submission.

Let us use the `ezplot` command to draw the graph of the equation

$$x^2 + y^2 = 25 \tag{1}$$

You do this by entering:

```
>> ezplot('x^2 + y^2 = 25', [-6, 6], [-6, 6])
```

Look at your picture. It doesn't look like a circle, does it? You can improve its look by entering:

```
>> axis square
```

Clearly, the graph you see is not the graph of a function. However, let us see what happens if we zoom in on a neighborhood of the point $(3, 4)$. Enter:

```
>> axis([1 5 2 6])
```

As you can see, the graph now looks pretty much like the graph of a function. In class we found the equation of the tangent line to the graph of this function. It is:

$$y = -\frac{3}{4}(x - 3) + 4 \tag{2}$$

Equivalently, this equation can be written as:

$$-\frac{3}{4}(x - 3) + 4 - y = 0 \tag{3}$$

Let us plot the tangent line on the same graph. Enter:

```
>> hold on
>> ezplot('-(3/4)*(x-3) + 4 - y = 0', [-6, 6], [-6, 6])
```

You should see a nice tangent line to the graph of a function. Let us zoom out again to see how the tangent line is related to the whole graph of equation (1). Enter:

```
>> axis([-6 6 -6 6])
```

Now give a descriptive title to your picture and print it if submission is required.

It is not the case that for *all* points (x_0, y_0) on the graph of equation (1) the equation defines a function $y(x)$ in a neighborhood of (x_0, y_0). Please mark on your printout all points (x_0, y_0) where equation (1) *does not* define such a function.

Now let us explore the equation:

$$x^3 + y^3 = 4.5xy \tag{4}$$

To clean your graphics window, enter:

```
>> hold off
```

Now enter:

```
>> ezplot('x^3 + y^3 - 4.5*x*y = 0', [-2, 4], [-2, 4])
```

The shape you see in the picture is called the "folium of Descartes." The whole figure clearly is not the graph of a function. To see what happens in a neighborhood of the point $(1, 2)$, enter:

```
>> axis([0.5 1.5 1.5 2.5])
```

Again, you can see the graph of a function. In the handout, we computed the equation of the tangent line to the graph of this function at $(1, 2)$. It is:

$$y = \frac{6}{7.5}(x - 1) + 2 \tag{5}$$

Add the graph of this tangent line to the picture by entering:

```
>> hold on
>> ezplot('(6/7.5)*(x-1) + 2 - y = 0', [-2, 4], [-2, 4])
```

Now zoom out again to look at the whole picture. Enter:

```
>> axis([-2 4 -2 4])
```

Add a suitable title to your picture and print it if submission is required.