Math 640B **Winter 2004**

Homework 5, due Friday 13 February.

1. (30 points) Section 6.12 problem 4.
   Do this problem as a Good Problem.

2. (50 points) In Section 6.12 we learned that the sines/ cosines (or complex exponentials) are orthogonal both as functions using the continuous inner product and as vectors using a discrete inner product. Then in Section 6.13 we learned how to expand using a fast transform.

   In Section 6.8 example 3 we learned that the Chebyshev polynomials are orthogonal as functions using a different continuous inner product.

   (a) What discrete inner product do we need to use to make the Chebyshev polynomials orthogonal as vectors? At which points do we need sample values of our input function? What sums do we need to compute to get the coefficients of a vector expanded in the (discrete) Chebyshev polynomials? (These three are really the same question.)

   (b) Write a program that implements a *fast* discrete Chebyshev transform, one that is $\mathcal{O}(N \log N)$ like the fft. Your program should consist of some sort of preprocessing, then a call to a built-in fft routine (e.g. in Matlab), and then some post-processing.

3. (20 points) Compute the Haar wavelet transform of the vector

$$(1, 3, -2, -2, -5, 6, 7, 0).$$