Math 163B **Study guide for Final Exam** **Winter 2003**

- I will have office hours Thursday 11am-12, Friday 10-11am, Monday 3-4pm, Tuesday 2-3pm, and Wednesday 10-11am and 3-4pm. You can also make an appointment to see me at another time.
- There are a lot of formulas to remember. If you forget one, you can "buy" it during the exam for a few points. Generally it is worth buying a formula if it will enable you to do the second half of the problem and earn points there.

About 25% of the final will be from section 8.1, since we have not had a test on that section.

About 50% will be questions very similar to those that appeared on the earlier tests.

The remaining 25% will be synthesis or concept questions. They may require you to mix techniques from different sections, or apply what you have learned in new contexts.

Notes

Sections 2.4, 2.5, 2.6: Be able to graph exponentials and logs and use their properties. Know how to do interest with the various types of compounding, and exponential growth of populations. You can ignore the definition of e, how to change the base of a logarithm, the rules of 70 and 72, effective rates, present value, and applications like radioactive decay and cooling.

Sections 4.4 and 4.5: Be able to do derivatives involving exponentials and logarithms. This means you also need to remember the chain, product, and quotient rules.

Sections 7.1 and 7.2: Be able to take antiderivatives, using substitution when needed.

Section 7.3: Be able to approximate the definite integral using a few rectangles. Be able to interpret what it means and how it relates to area.

Sections 7.4 and 7.5: Be able to apply the fundamental theorem of Calculus to solve definite integrals. Be able to find the area between a function and the axis, or the area between two curves.

Section 8.1: Be able to do integration by parts.

Sample synthesis and concept questions

1. So far we do not know how to evaluate $\int_0^3 \sqrt{9-x^2}\,dx$. Interpret this integral in geometrical terms, and use your interpretation to find the value of the integral.

2. Suppose f is a continuous, increasing function on $[a,b]$. Find lower and upper bounds on $\int_a^b f(x)dx$ in terms of a, b, $f(a)$, and $f(b)$. (Hint: Draw a picture.)

3. Suppose f and g are continuous functions with the following properties:

$f(0) = 2$	$f(1) = 0$	$f(2) = 1$
$g(0) = 1$	$g(1) = 2$	$g(2) = 0$
$\int_0^1 f(x)dx = \pi$	$\int_1^2 f(x)dx = \pi^3$	$\int_2^3 f(x)dx = \pi^5$
$\int_0^1 g(x)dx = \sqrt{2}$	$\int_1^2 g(x)dx = \sqrt{3}$	$\int_2^3 g(x)dx = \sqrt{5}$
$f'(0) = e$	$f'(1) = e^3$	$f'(2) = e^5$
$g'(0) = \sqrt{7}$	$g'(1) = \sqrt{11}$	$g'(2) = \sqrt{13}$

Evaluate the following. If one cannot be evaluated with the given information, write "NOT ENOUGH INFORMATION."

(a) $\int_3^2 g(x)dx$

(b) $\int_1^2 (5f(x) + g(x))dx$

(c) $\int_0^1 f(x)g(x)dx$

(d) $\int_1^2 f(g(x))dx$

(e) $\int_2^3 f(g(x))f'(x)dx$

(f) $\int_1^0 f(g(x))g'(x)dx$

(g) $\frac{d}{dy}g(y)$ at $y = 2$

(h) $\frac{d}{dx}(f(x)g(x))$ at $x = 0$

(i) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$ at $x = 0$

(j) $\frac{d}{dx}f(g(x))$ at $x = 1$

(k) $\int_0^2 f'(r)\,dr$

(l) $\int_6^6 f''(x)\,dx$

(m) $\int_0^2 f(g'(x))\,dx$

4. Interpret

$$\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^{3/2} + \left(\frac{2}{n}\right)^{3/2} + \cdots + \left(\frac{n}{n}\right)^{3/2}\right]$$

as a definite integral and solve this integral to find the numerical value of the limit.

5. (a) How are 2^x, 2^{-x}, $(1/2)^x$, and $(1/2)^{-x}$ related?
   (b) How are 2^x and $\log_2(x)$ related?
   (c) Compare and contrast 2^x and x^2.