Vaccination Strategies for Small Worlds

Winfried Just
Department of Mathematics, Ohio University

Joint work with Hannah Callender
University of Portland

BEER 2016, Charleston, SC
October 15, 2016

Outline of the talk

This talk will describe a recently submitted joint book chapter with Hannah Callender.

1. Preach to the choir about the need to create teaching materials for REUs.
2. Talk about network-based models of disease transmission. Cover the basics that the audience already knows.
3. Spring the big non-surprise on the kind of projects in a chapter titled "Vaccination Strategies for Small Worlds."

Can I really do this to all the wonderful people in my audience???

I'll let others do the talking.

Bob: Have you read Dean Howard's article in the Post about Research Experiences for Undergraduates?

This would be a great thing to have on the CV.

Cindy: Yes! This sounds fun and exciting!! Let's do one!!!

Alice: But how to find an instructor who would direct it?

Bob: Let's ask Professor Drumpf. He is always up for anything that will make the news.

Ohio University – Since 1804 Department of Mathematics

Looking for an instructor

Cindy: Professor Drumpf, would you be so kind and direct an REU project for us?

Prof. Drumpf: Absolutely!! I will give you a hands-on research experience so good that you will keep talking about it for decades!

Bob and Cindy: (Stunned.)

Alice: What will be the topic?

(It dawns on Prof. Drumpf that this would require some actual expertise.)

Bob: And how much work will be involved?

Prof. Drumpf: Here is the deal: You sign up for the project with me. My colleague Dr. Penny will then fill you in on the details and meet with you every week to discuss your progress. Go see him at 1:30. (Dismisses the students and calls Dr. Penny.)

Dr. Penny's problem

(Upon hearing the news, Dr. Penny walks right into the office of his Department Chair.)

Dr. Penny: I have been approached to direct an REU for three students. Will be happy to do it if you count it towards my teaching load.

Chair Wood: We cannot do this. Budget cuts. Sabbaticals. . . .

Dr. Penny: (Cuts her off.) But the Dean is all in favor and it will look good for the department!! The prep time alone involved in designing such a project is comparable with the work I normally put into teaching one class!

Chair Wood: Couldn't you do it based on some published materials that give the needed background and then outline some open-ended research-level projects?

Dr. Penny: There aren't any such materials in my research area!

Dr. Penny meets with the students

Dr. Penny: Prof. Drumpf needs to cancel the plan.

Alice, Bob, Cindy: But why???

Dr. Penny: Because the system is rigged!

(Dismisses the students.)

Bob: Bummer! But let's try some other instructor.

(The students make a few more attempts;
always with the same result.)

Alice: There is one faculty member left . . .

Bob: Are you kidding? This guy expects his students to do an insane amount of work!

Cindy: Couldn't we at least ask him and then decide later?

The first meeting with "this guy"

Alice: Dr. Just, what do you do in terms of mathematical research?

WJ: I play.

Cindy: You mean, like, with toys????

WJ: Yes. My toys are called mathematical models.

Bob: And you get paid for this???

WJ: Yes. Like children's toys, my models can teach us how the real world works.

Alice: Are you serious??

Cindy: (Senses an opening.) You know, we are really here to ask you whether we could play with your toys.

WJ: Yes you can!! Of course!

Alice: We mean, as an REU project.

Bob: For credit.

Mathematical Epidemiology

(A moment of heavy silence.)

WJ: We could do a project on mathematical epidemiology perhaps.

Cindy: Is this like . . . mathematical medicine?

WJ: (Regains his composure and goes into lecturing mode:)

Medicine studies the causes, symptoms, and progression of infectious diseases.

Its goal is to cure diseases.

Epidemiology studies how infections spread.

Its goal is to find ways to prevent or at least limit the spread of infections.

The latter can be achieved by appropriate control measures:

- vaccination,
- quarantine,
- behavior modifications (for human diseases).

Alice: But what's math got to do with it?

WJ: (Happy to continue lecturing:)

In order to prevent infections, we need to understand how they spread.

But real life is complicated, messy, rich in meanings.

If we were to pay attention to everything at once, we would be in a mental fog.

If we want to understand what really goes on, we need to focus on the essentials and ignore that there is more.

Mathematical models do just that: They are toy worlds that ignore most of the messy details, cut through the fog, and reduce a situation to its driving mechanisms.

What's love got to do with it? by Tina Turner

Tina: This is exactly as in my song!!! Listen:

You must understand how the touch of your hand . . .
I've been thinking about my own protection . . .
Who needs a heart when a heart can be broken . . .

Tina: To prevent heartbreak, I need to understand what drives the situation.

Acting confused when you're close to me
If I tend to look dazed . . .

Tina: I'm in a mental fog and cut right through it:

Opposites attract. It's physical, only logical.
You must try to ignore that it means more than that . . .

Tina and WJ: That's exactly how mathematical models work!

So what's math got to do with it?

Cindy: So, mathematical epidemiology would be, like, spreading a "second-hand infection" inside a mathematical model?

WJ: So to speak, yes.

Cindy: And nobody gets hurt by it?

WJ: Absolutely nobody. Most importantly, without causing any actual suffering, we can explore many different options for control measures, like, for example, different policies for vaccinating people.

Alice: To see which one works best?

WJ: Right.

Alice: And then recommend that the best option be implemented?

WJ: That's exactly the point. You have put it in a nutshell.

Alice and Cindy: Wow!!

Yes, but ...

Bob: Control measures may be costly though. For example, not enough vaccine may be available to protect the whole population.

WJ: Good point. This may be a problem.

Bob: So "best" would mean the strategy that will be expected to give the largest reduction in suffering among all options that are economically affordable?

WJ: I couldn't have said it any better.

Alice: But you need to make sure your models are telling you the correct thing, or else real people might suffer!!!

WJ: (Seizes the opportunity to lecture again.)
This is why we need to be very precise and careful about setting up our models.

Epidemic models in a nutshell: Some terminology

Infectious diseases that are triggered when pathogens such as viruses or bacteria enter a host (human, animal, plant).

We study how the infection spreads between hosts of a given population.

Let's focus on immunizing infections that spread by direct contact between hosts.

Then we can partition the population into The S-, I-, and R-compartments are the sets of hosts that are susceptible, infectious, and removed, respectively.

The spread of the infection is modeled as movement (of sorts) of hosts from one compartment into another.

An example of an ODE-based SIR-model

For an actual model we need:

- Variables: s, i, r They will represent the proportions of hosts in the S-, I-, and R-compartments, respectively.
- A state space: Each of the variables can take values in $[0, 1]$ and they must add up to 1.
- Some parameters: α, β. Positive constants whose values need to be empirically determined.
- A rule of change: This governs the change of the variables over time.

$$
\begin{aligned}
\frac{ds}{dt} &= -\beta\, i\, s, \\
\frac{di}{dt} &= \beta\, i\, s - \alpha\, i, \\
\frac{dr}{dt} &= \alpha\, i.
\end{aligned}
$$

Is this fun? Is this interesting?

Cindy: Your model doesn't look much fun to play with.

WJ: It makes some very interesting predictions though.

Alice: Can you give us an example?

WJ: The model predicts, for example, that the final size of any major outbreak that is caused by one index case in an otherwise susceptible population only depends on the ratio $R_0 = \frac{\beta}{\alpha}$, which is the so-called "basic reproductive ratio." For example, when $R_0 = 1.5$, then 58.28% of the entire population will experience infection.

Does this make sense?

Alice: This doesn't make any sense to me.

WJ: Good.

Alice: Are you making fun of me?

WJ: No. Tell me why it doesn't make sense to you.

Alice: Because, as you said, if all parameters are fixed, then the predicted percentage is also fixed. If it happens to be 58.28%, then the model would supposedly predict that out of a population of 10,000 hosts, always exactly 5,828 will experience infection. This is nonsense!!!

WJ: And why, exactly, would this be nonsense?

Alice: Stop making fun of me!!

Everybody in the audience will tell you why!

Agent-based models

WJ: (Lecturing again.) What you are saying is that disease transmission is a stochastic process.

In the REU we would explore agent-based models that take into account the inherent stochasticity.

For example, an agent-based version of the model that I just showed you that approximately 58% of the population will experience infection in a major outbreak.

This will be true if $R_0 = 1.5$ and we assume uniform mixing, which means that every host is equally likely to make contact with every other host.

Alice: This cannot be true!!

WJ: It will be approximately true for populations of animals that move around a lot, encounter each other rarely, and have no social or territorial structure.

Contact networks

Alice: But not for human populations! Think about your chances of meeting face-to-face with your next-door neighbor or with President Obama!!

WJ: I see your point. So how would you model the social and territorial structure of human contacts?

Alice: We make contact mostly with the people in our contact network, right?

WJ: Yeah ...

Alice: So you need to assume that transmission between two hosts can occur only when these hosts are connected in the contact network.

Network-based models

WJ: If I will make this assumption and still get the same predictions as under the uniform mixing assumption, will you then trust the predictions of the simpler model?

Alice: A little more, but there may still be something else that the model overlooks. In any case, I bet you don't get the same predictions, and in that case the first model is too simplistic to base real health care policy on it!!

WJ: We can model contact networks as graphs and assume that the disease can be transmitted only between two hosts that are connected by an edge.

Cindy: So you can think of two people connected by an edge as friends?

WJ: This will give you a good intuition about this concept.

Cindy: Do Facebook friends count?

Alice: Nope! Well, unless you study the spread of computer viruses. Or of gossip, for that matter.

An REU project?

WJ: If you are interested in this topic, we could do an REU on network-based models of disease transmission.

Alice: What, exactly, would this involve?

WJ: Simulations with a software tool **IONTW**, deriving conjectures from the observed outcomes, and trying to prove some of these conjectures.

Cindy: Cool!

Bob: How much does a copy of **IONTW** cost?

WJ: You can download it for free from

`http://www.ohio.edu/people/just/IONTW/`

`https://qubeshub.org/iontw`

What, exactly, would we do?

Alice: Would your project involve some actual open research problems?

WJ: Possibly.

Cindy: And how could we learn all the needed mathematical background?

WJ: You would first work through the many exercises in the modules to learn the concepts and get some experience before attempting real open research problems.

Bob: This is too much material for a one-semester project.

WJ: Let's all think about it and meet again next week.

WJ: (To himself:) Now what do I do?

(Checks his e-mail.)

Hannah to the rescue

(Sees a new message from Hannah Callender.)

HC: Hey Vinny, how would you like us to contribute a chapter to the volume

Foundations for Undergraduate Research: A Primer for Starting Student Projects.
A. Wootton, V. Peterson, C. Lee, eds., Springer Verlag?

Let's include a sequence of selected relevant exercises and lecture-style material from some of our modules.

This would build up the know-how for attempting some open-ended research projects.

Any suggestions for the projects?

Vaccinating small-world networks

WJ: This is exactly what these students need!

The projects focus on small-world networks, where the distances between two nodes tend to be small (the so-called small-world property), and clustering coefficients tend to be high. The latter roughly means that two hosts that share a common neighbor in the contact network are more likely to be connected by an edge than two randomly chosen hosts.

More precisely, our projects study optimal vaccination strategies in these models, that is, methods for choosing sets of hosts of a given limited size that will most likely reduce the probability of a major outbreak or its final size to the largest possible extent.

Students are asked to explore how the effectiveness of certain such strategies depends on the model parameters, both with simulations and analytically.

The existing literature on these problems is rather limited, and at least some new results should be within reach of students.

About the characters in this play

The parts of WJ and HC are modeled on the authors of the book chapter described in these slides.

None of the other characters is based on a particular student or faculty member. The lines spoken by them and the plot are purely imaginary.