Typical values of the Reynolds number for various animals moving through air or water are listed below. For which cases is inertia of the fluid important? For which cases do viscous effects dominate? For which cases would the flow be laminar, turbulent? Explain.

<table>
<thead>
<tr>
<th>Animal</th>
<th>Speed</th>
<th>Re</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) large whale</td>
<td>10 m/s</td>
<td>300,000,000</td>
</tr>
<tr>
<td>(b) flying duck</td>
<td>20 m/s</td>
<td>300,000</td>
</tr>
<tr>
<td>(c) large dragonfly</td>
<td>7 m/s</td>
<td>30,000</td>
</tr>
<tr>
<td>(d) invertebrate larva</td>
<td>1 mm/s</td>
<td>0.3</td>
</tr>
<tr>
<td>(e) bacterium</td>
<td>0.01 mm/s</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

**Inertia important if** \( \text{Re} \approx 1 \) (i.e. whale, duck, dragonfly)

**Viscous effects dominate if** \( \text{Re} \approx 1 \) (i.e. larva, bacterium)

Boundary layer flow becomes turbulent for \( \text{Re} \) on the order of \( 10^5 \) to \( 10^6 \) (i.e. whale and perhaps the duck)

The flow would be laminar for the dragonfly, larva, and bacterium and perhaps the duck.
9.13 A viscous fluid flows past a flat plate such that the boundary layer thickness at a distance 1.3 m from the leading edge is 12 mm. Determine the boundary layer thickness at distances of 0.20, 2.0, and 20 m from the leading edge. Assume laminar flow.

For laminar flow \( \delta = C \sqrt{x} \), where \( C \) is a constant.

Thus,
\[
C = \frac{\delta}{\sqrt{x}} = \frac{12 \times 10^{-3} \text{ m}}{\sqrt{1.3 \text{ m}}} = 0.0105 \text{ or } \delta = 0.0105 \sqrt{x} \text{ where } x \sim m, \delta \sim m
\]

<table>
<thead>
<tr>
<th>( x, m )</th>
<th>( \delta, m )</th>
<th>( \delta, \text{ mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.00470</td>
<td>4.70</td>
</tr>
<tr>
<td>2.0</td>
<td>0.0148</td>
<td>14.8</td>
</tr>
<tr>
<td>20.0</td>
<td>0.0470</td>
<td>47.0</td>
</tr>
</tbody>
</table>

9.14 If the upstream velocity of the flow in Problem 9.13 is \( U = 1.5 \text{ m/s} \), determine the kinematic viscosity of the fluid.

For laminar flow \( \delta = 5 \sqrt{\frac{Vx}{U}} \), or \( \nu = \frac{U \delta^2}{25x} \)

Thus,
\[
\nu = \frac{(1.5 \text{ m/s})(12 \times 10^{-3} \text{ m})^2}{25 (1.3 \text{ m})} = 6.65 \times 10^{-6} \text{ m}^2/\text{s}
\]
9.16 Approximately how fast can the wind blow past a 0.25-in.-diameter twig if viscous effects are to be of importance throughout the entire flow field (i.e., Re < 1)? Explain. Repeat for a 0.004-in.-diameter hair and a 6-ft-diameter smokestack.

\[ Re = \frac{U D}{v} < 1 \quad \text{or} \quad U < \frac{v}{D} \] if viscous effects are to be important throughout the flow.

For standard air \( v = 1.57 \times 10^{-5} \text{ ft}^2/\text{sec} \),

Thus,

\[ U < \frac{1.57 \times 10^{-4}}{D}, \] where \( D \) is the diameter in feet.

<table>
<thead>
<tr>
<th>object</th>
<th>( D, \text{ ft} )</th>
<th>( U, \text{ ft/sec} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>twig</td>
<td>2.08 \times 10^{-5}</td>
<td>7.54 \times 10^{-3}</td>
</tr>
<tr>
<td>hair</td>
<td>3.33 \times 10^{-4}</td>
<td>0.471</td>
</tr>
<tr>
<td>smokestack</td>
<td>6</td>
<td>2.62 \times 10^{-5}</td>
</tr>
</tbody>
</table>
9.22 An atmospheric boundary layer is formed when the wind blows over the earth's surface. Typically, such velocity profiles can be written as a power law: \( u = ay^n \), where the constants \( a \) and \( n \) depend on the roughness of the terrain. As is indicated in Fig. P9.22, typical values are \( n = 0.40 \) for urban areas, \( n = 0.28 \) for woodland or suburban areas, and \( n = 0.16 \) for flat open country (Ref. 23). (a) If the velocity is 20 ft/s at the bottom of the sail on your boat \( (y = 4 \text{ ft}) \), what is the velocity at the top of the mast \( (y = 30 \text{ ft}) \)? (b) If the average velocity is 10 mph on the tenth floor of an urban building, what is the average velocity on the sixtieth floor?

\[
(u) \quad u = Cy^{0.16}, \text{ where } C \text{ is a constant} \\
\quad \text{Thus, } \frac{u_2}{u_1} = \left(\frac{y_2}{y_1}\right)^{0.16} \quad \text{or } u_2 = 20 \text{ ft/s} \left(\frac{30 \text{ ft}}{4 \text{ ft}}\right)^{0.16} = 27.6 \text{ ft/s} \\
\]

\[
(b) \quad \bar{u} = Cy^{0.40}, \text{ where } C \text{ is a constant} \\
\quad \text{Thus, } \frac{u_2}{u_1} = \left(\frac{y_2}{y_1}\right)^{0.40} \quad \text{or } \bar{u}_2 = 10 \text{ mph} \left(\frac{60 \text{ ft}}{10 \text{ ft}}\right)^{0.4} = 20.5 \text{ mph} \\
\]
9.45 A sphere of diameter \( D \) and density \( \rho_s \), falls at a steady rate through a liquid of density \( \rho \) and viscosity \( \mu \). If the Reynolds number, \( Re = \rho DU/\mu \), is less than 1, show that the viscosity can be determined from \( \mu = gD^2(\rho_s - \rho)/18 \, U \).

For steady flow \( \sum F_x = 0 \) or \( D + F_B = W \), where \( F_B = \) buoyant force = \( \rho_l V = \rho g (\frac{4}{3}) \pi (\frac{D}{2})^3 \)

\( W = \) weight = \( \rho_l g V = \rho g (\frac{4}{3}) \pi (\frac{D}{2})^3 \)

and \( D = \) drag = \( C_D \frac{1}{2} \rho U^2 D^2 \), or since \( Re < 1 \)

\( D = 3\pi DU \mu \)

Thus,

\( 3\pi DU \mu + \rho g (\frac{4}{3}) \pi (\frac{D}{2})^3 = \rho_s g (\frac{4}{3}) \pi (\frac{D}{2})^3 \)

which can be rearranged to give

\( \mu = \frac{gD^2(\rho_s - \rho)}{18 \, U} \)
9.46 The square flat plate shown in Fig. P9.46a is cut into four equal-sized pieces and arranged as shown in Fig. P9.46b. Determine the ratio of the drag on the original plate [case (a)] to the drag on the plates in the configuration shown in (b). Assume laminar boundary flow. Explain your answer physically.

For case (a):
\[ \frac{\Delta F_a}{64} = \frac{1}{2} \rho U^2 C_{DF} A \]
where \( C_{DF} = \frac{1.328}{\sqrt{Re_U}} = \frac{1.328}{\sqrt{\frac{U}{V}}} \) and \( A = \ell^2 \)

Thus,
\[ \Delta F_a = \frac{1}{2} \rho U^2 \frac{1.328 \sqrt{\ell^2}}{\ell} \ell^2 = 0.664 \rho U^{\frac{3}{2}} V \ell^2 \]  

(1)

For case (b):
\[ \frac{\Delta F_b}{64} = \frac{1}{2} \rho U^2 C_{DF} A \]
where \( C_{DF} = \frac{1.328}{\sqrt{Re_U}} \) and \( A = \left(\frac{4\ell}{\ell}\right) \ell^2 = \ell^2 \)

Thus,
\[ \Delta F_b = \frac{1}{2} \rho U^2 \frac{1.328 \sqrt{\ell^2}}{\ell} \ell^2 = \frac{1}{2} \left(0.664 \rho U^{\frac{3}{2}} V \ell^2 \right) \]  

(2)

By comparing Eqs. (1) and (2) we see that
\[ \frac{\Delta F_a}{\Delta F_b} = 2.0 \]

In case (b) the boundary layer on the rear plate is thicker than on the front plate. Hence the shear stress is less on the rear plate than it is on that plate in configuration (a), giving less drag for case (b) than for case (a), even though the total areas are the same.
9.5D A rectangular car-top carrier of 1.6-ft height, 5.0-ft length (front to back), and 4.2-ft width is attached to the top of a car. Estimate the additional power required to drive the car with the carrier at 60 mph through still air compared with the power required to driving only the car at 60 mph.

\[ D = C_0 \frac{1}{2} \rho U^2 A \quad \text{and} \quad P = Ud \quad \text{power} \]

From Fig. 9.31 with \[ \frac{f}{A} = \frac{5.0}{14.4} = 0.35 \] we obtain \( C_0 = 1.3 \)

Hence,

\[ D = 1.3 \left( \frac{1}{2} \right) (0.00238 \text{ slug/s}) (14.4 \text{ ft})(4.2 \text{ ft})(80.5 \text{ lb}) = 80.5 \text{ lb} \]

Thus, from Eq. (1),

\[ P = (88 \frac{\text{ lb}}{\text{ ft}})(80.5 \text{ lb}) \frac{1 \text{ hp}}{550 \text{ ft-lb}} = 12.9 \text{ hp} \]
A 22 in. by 34 in. speed limit sign is supported on a 3-in. wide, 5-ft-long pole. Estimate the bending moment in the pole at ground level when a 30-mph wind blows against the sign. (See Video V9.9.) List any assumptions used in your calculations.

For equilibrium, \( \sum M_0 = 0 \) or

\[
M_B = 2.5 \text{ ft} \cdot \Delta P + (5 + \frac{17}{2}) \text{ ft} \cdot \Delta S,
\]

where \( \Delta P \) = drag on the pole and \( \Delta S \) = drag on the sign.

From Fig. 9.28 with \( R/D < 0.1 \) for the sign,

\[ C_{DS} = 1.9 \]

From Fig. 9.19 if the part acts as a square rod with sharp corners \( C_{DP} = 2.2 \). Thus, with \( U = 30 \text{ mph} = 44 \text{ ft/s} \),

\[
\Delta S = \frac{1}{2} \rho U^2 C_{DS} A_S = \frac{1}{2} (0.00238 \frac{\text{lb}}{\text{ft}^2}) (44 \text{ ft/s})^2 (1.9) \left( \frac{22 \text{ (34)} \text{ ft}^2}{744} \right) = 22.7 \text{ lb}
\]

and

\[
\Delta P = \frac{1}{2} \rho U^2 C_{DP} A_P = \frac{1}{2} (0.00238 \frac{\text{lb}}{\text{ft}^2}) (44 \text{ ft/s})^2 (2.2) \left( \frac{\frac{3}{4} (5) \text{ ft}^2}{744} \right) = 6.3 \text{ lb}
\]

Thus, from Eq. (1):

\[
M_B = 2.5 \text{ ft} \cdot (6.3 \text{ lb}) + (5 + \frac{17}{2}) \text{ ft} \cdot (22.7 \text{ lb}) = 162 \text{ ft-lb}
\]
9.61 Determine the moment needed at the base of 20-m-tall, 0.12-m-diameter flag pole to keep it in place in a 20 m/s wind.

For equilibrium, \( M = \frac{1}{2} D \) where

\[
D = C_d \frac{1}{2} \rho U^2 l D
\]

Since \( Re = \frac{U D}{\nu} = \frac{20 \frac{m}{s})(0.12\, m)}{1.44 \times 10^{-5} \frac{m^2}{s}} = 1.54 \times 10^6 \), it follows from Fig. 2.21 that \( C_d = 1.2 \)

Thus, \( D = 1.2 \left( \frac{1}{2} \right)(1.23 \frac{kN}{m^3})(20 \frac{m}{s})(20\, m)(0.12\, m) = 708\, N \)

Hence, from Eq. (1)

\[
M = \frac{20\, m}{2} (708\, N) = 7080\, N/m
\]
9.69 As shown in Video V9.7 and Fig. P9.69 a vertical wind tunnel can be used for skydiving practice. Estimate the vertical wind speed needed if a 150-lb person is to be able to "float" motionless when the person (a) crouches up as in a crouching position or (b) lies flat. See Fig. 9.30 for appropriate drag coefficient data.

For equilibrium conditions

\[ W = \rho C_d A U^2 \]

Assume \( W = 160 \text{ lb} \) and \( C_d A = 9 \text{ ft}^2 \) (see Fig. 9.30).

Thus,

\[ 160 \text{ lb} = \left( \frac{1}{2} \right) (0.00236 \text{ slugs/ft}^3)(9 \text{ ft}^2) \left( \frac{1 \text{ mph}}{1 \text{ ft/s}} \right) \text{ where } U = \frac{4}{3} \text{ ft/s} \]

or

\[ U = \left( \frac{122 \text{ ft}}{s} \right) \left( \frac{1 \text{ mph}}{5280 \text{ ft}} \right) \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 83.2 \text{ mph} \]

Note: If the skydiver "crouched up into a ball", then \( C_d A = 2.5 \text{ ft}^2 \) (see Fig. 9.30) and \( U = 158 \text{ mph} \).
9.74 Estimate the wind force on your hand when you hold it out of your car window while driving 55 mph. Repeat your calculations if you were to hold your hand out of the window of an airplane flying 550 mph.

\[ F = C_D \frac{1}{2} \rho U^2 A, \quad \text{where} \quad U = (55 \text{ mph}) \left( \frac{88 \text{ ft}}{60 \text{ mph}} \right) = 80.7 \frac{\text{ft}}{\text{s}} \]

Assume your hand is 4 in. by 6 in. in size and acts like a thin disc with \( C_D = 1.1 \) (see Fig. 9.29).

\[ \frac{F}{(1.1)(\frac{1}{2})(0.00236)(80.7 \frac{\text{ft}}{\text{s}})^2}{(1.5 \text{ ft})(0.5 \text{ ft})} = 1.42 \text{ lb} \]

Thus, the lift force is zero.

For \( U = 550 \text{ mph} = 807 \frac{\text{ft}}{\text{s}} \) (i.e., a 10 fold increase in \( U \)) the drag will increase by a factor of 100 (i.e., \( d^2 \sim U^2 \)), or \( F = 142 \text{lb} \)

Note: We have assumed that \( C_D \) is not a function of \( U \). That is, it is not a function of either \( Re = \frac{Ud}{v} \) or \( Ma = \frac{U}{a} \).
The United Nations Building in New York is approximately 87.5-m wide and 154-m tall. (a) Determine the drag on this building if the drag coefficient is 1.3 and the wind speed is a uniform 20 m/s. (b) Repeat your calculations if the velocity profile against the building is a typical profile for an urban area (see Problem 9.22) and the wind speed halfway up the building is 20 m/s.

(a) \[ D = C_d \frac{1}{2} \rho U^2 A = 1.3 \left( \frac{1}{2} \right) (1.23 \text{ m}) (20 \text{ m/s})^2 (154 \text{ m})(875 \text{ m}) \]

or

\[ D = 4.21 \times 10^6 \text{ N} \approx 4.21 \text{ MN} \]

(b) For an urban area, \( U = C y^{0.4} \).

Thus, with \( U = 20 \text{ m/s} \) at \( y = \frac{h}{2} = 77 \text{ m} \).

we obtain

\[ C = \frac{20}{77^{0.4}} = 3.52 \], or \( U = 3.52 \cdot y^{0.4} \) with \( U \sim y \), \( y \sim m \).

The total drag is

\[ D = \int_{y_0}^{y_{154}} dD = \int C_d \frac{1}{2} \rho U^2 \, dA = \frac{1}{2} \rho C_d \left[ (3.52 \cdot y^{0.4})^2 (875 \text{ m}) \right] dy \]

or

\[ D = \frac{1}{2} (1.23)(1.3)(3.52)^2 (875 \text{ m}) \int_0^{154} y^{0.8} \, dy = 867 \left( \frac{1}{1.8} \right)(154)^{1.8} = 4.17 \times 10^6 \text{ N} \]

Thus,

\[ D = 4.17 \text{ MN} \]
9.77 Show that for level flight at a given speed, the power required to overcome aerodynamic drag decreases as the altitude increases. Assume that the drag coefficient remains constant. This is one reason why airlines fly at high altitudes.

For level flight $L = W$, where $W =$ airplane weight = constant and $L = C_D \frac{1}{2} \rho U^2 A$

If $U$ is to remain constant, then $C_D$ must increase as $\rho$ decreases (i.e., altitude increases).

Also, $P = \rho U$, where $P = C_D \frac{1}{2} \rho U^2 A$ or $P = C_D \frac{1}{2} \rho U^3 A$. For constant $U$, $C_D$, and $A$, the power decreases as altitude increases ($\rho$ decreases).
9.92 A rectangular wing with an aspect ratio of 6 is to generate 1000 lb of lift when it flies at a speed of 200 ft/s. Determine the length of the wing if its lift coefficient is 1.0.

Aspect ratio, \( A = \frac{b^2}{c} = 6 \)

= \( \frac{b}{c} \) for rectangular wing

The lift coefficient is given by,

\[
C_L = \frac{\frac{L}{\frac{1}{2} \rho v^2}}{A}
\]

\[
L = C_L \cdot \frac{1}{2} \rho v^2 (6c^2)
\]

1000 lb = 1.0 \( (\frac{1}{2}) (0.00238 \text{slug} \cdot \text{in}^2) (200 \text{ ft/s})^2 (6c^2) \)

\( 6c^2 = 21.0 \)

\( c = 1.87 \text{ ft} \)

\( b = 6(c) = 6(1.87 \text{ ft}) \)

\( b = 11.2 \text{ ft} \)
9.94 A Piper Cub airplane has a gross weight of 1750 lb, a cruising speed of 115 mph, and a wing area of 179 ft². Determine the lift coefficient of this airplane for these conditions.

For equilibrium \( L = W = 1750 \text{ lb} \), where \( L = C_L \frac{1}{2} \rho U^2 A \).

Thus, with \( U = (115 \text{ mph}) \frac{(60 \text{ mph})}{(60 \text{ mph})} = 169 \frac{\text{ft}}{\text{s}} \)

\[
C_L = \frac{L}{\frac{1}{2} \rho U^2 A} = \frac{1750 \text{ lb}}{\frac{1}{2} (0.00238 \frac{\text{sli}}{\text{ft}^3})(169 \frac{\text{ft}}{\text{s}})^3(179 \text{ ft}^2)} = 0.288
\]

9.95 A light aircraft with a wing area of 200 ft² and a weight of 2000 lb has a lift coefficient of 0.40 and a drag coefficient of 0.05. Determine the power required to maintain level flight.

For equilibrium \( L = W = 2000 \text{ lb} = C_L \frac{1}{2} \rho U^2 A \)
or \( 2000 \text{ lb} = (0.40) \frac{1}{2} (0.00238 \frac{\text{sli}}{\text{ft}^3}) U^2 (200 \text{ ft}^2) \)

Hence,

\[
U = 145 \frac{\text{ft}}{\text{s}}
\]

Also, \( \mathcal{D} = \text{power} = \mathcal{D} U \), where

\[
\mathcal{D} = C_D \frac{1}{2} \rho U^2 A = (0.05) \frac{1}{2} (0.00238 \frac{\text{sli}}{\text{ft}^3})(145 \frac{\text{ft}}{\text{s}})^3(200 \text{ ft}^2) = 250 \text{ lb}
\]

Note: This value of \( \mathcal{D} \) could be obtained from

\[
\frac{W}{\mathcal{D}} = \frac{C_L}{C_D} = \frac{0.40}{0.05} = 8 , \text{ or } \frac{W}{\mathcal{D}} = \frac{2000 \text{ lb}}{250 \text{ lb}} = 8
\]

Thus,

\[
\mathcal{D} = 250 \text{ lb} (145 \frac{\text{ft}}{\text{s}})^3 = 3.63 \times 10^6 \frac{\text{ft} \cdot \text{lb} \cdot \text{s}^3}{\text{ft}^3} \left( \frac{1 \text{ hp}}{550 \frac{\text{ft} \cdot \text{lb} \cdot \text{s}}{\text{sec}}} \right) = 65.9 \text{ hp}
\]
9.48 A wing generates a lift \( L \) when moving through sea-level air with a velocity \( U \). How fast must the wing move through the air at an altitude of 10,000 m with the same lift coefficient if it is to generate the same lift?

\[ L = C_L \frac{1}{2} \rho U^2 \text{A} \quad \text{so with} \quad L, C_L, \text{and A constant} \]

\[ (\rho U^2)_{\text{sea level}} = (\rho U^2)_{10,000 \text{ m}} \]

Hence,

\[ U_{10,000 \text{ m}} = \left( \frac{\rho_{\text{sea level}}}{\rho_{10,000 \text{ m}}} \right)^{\frac{1}{2}} U_{\text{sea level}} = \left( \frac{1.23}{0.41} \right)^{\frac{1}{2}} U_{\text{sea level}} \]

or

\[ U_{10,000 \text{ m}} = 1.72 U_{\text{sea level}} \]
9.101 A Boeing 747 aircraft weighing 580,000 lb when loaded with fuel and 100 passengers takes off with an airspeed of 140 mph. With the same configuration (i.e., angle of attack, flap settings, etc.) what is its takeoff speed if it is loaded with 372 passengers. Assume each passenger with luggage weighs 200 lb.

For steady flight \( L = C_L \frac{1}{2} \rho U^2 A = W \) \hspace{2cm} (1)

Let \((\ )_{100}\) denote conditions with 100 passengers and \((\ )_{372}\) with 372 passengers. Thus, with \( C_{L_{100}} = C_{L_{372}} \), \( A_{100} = A_{372} \), and \( \rho_{100} = \rho_{372} \) Eq. (1) gives

\[
\frac{L_{100}}{L_{372}} = \frac{\frac{1}{2} \rho_{100} U_{100}^2}{\frac{1}{2} \rho_{372} U_{372}^2} \quad \text{or} \quad U_{100}^2 = \frac{T_{372} \left( 580,000 + (372 - 100)(200) \right) lb}{580,000 lb}, \quad \text{with} \ U_{100} = 140 \text{ mph}
\]

Thus, \( U_{372} = 146 \text{ mph} \)

9.102 Show that for unpowered flight (for which the lift, drag, and weight forces are in equilibrium) the glide slope angle, \( \theta \), is given by \( \tan \theta = C_D / C_L \).

For steady unpowered flight \( \sum F_x = 0 \) gives \( D = W \sin \theta \)

\( \sum F_y = 0 \) gives \( L = W \cos \theta \)

Thus,

\[
\frac{D}{L} = \frac{W \sin \theta}{W \cos \theta} = \tan \theta, \quad \text{where} \quad \tan \theta = \frac{1}{2} \frac{\rho U^2 A C_D}{\frac{1}{2} \rho U^2 A C_L} = \frac{C_D}{C_L}
\]

Hence, \( \tan \theta = \frac{C_D}{C_L} \)
9.107 Commercial airliners normally cruise at relatively high altitudes (30,000 to 35,000 ft). Discuss how flying at this high altitude (rather than 10,000 ft, for example) can save fuel costs.

For level flight \( W = \text{aircraft weight} = C_2 \pm \rho U^2 A \)

Thus, for given \( W, C_2 \), and \( A \) the dynamic pressure is constant, independent of altitude. That is

\[
\pm \rho U^2 \frac{10000}{10,000} \text{ ft} = \pm \rho U^2 \frac{40000}{10,000} \text{ ft}, \text{ or } U_{10,000} = \left( \frac{P_{10,000}}{P_{40,000}} \right)^{1/2} U_{40,000}
\]

Hence, \( U_{10,000} > U_{40,000} \)

Also, since the drag is \( D = C_D \pm \rho U^2 A \) it follows that

\[
D_{10,000} = C_D \pm \rho U_{10,000}^2, \quad \text{or } \quad D_{10,000} = C_D \pm \rho U_{40,000}^2
\]

Hence, the aircraft can fly faster at high altitudes with the same amount of drag \( (D_{10,000} = D_{40,000}) \)